Effects of experimental uncertainty on the calculation of hillslope flow paths

Mark D. Sherlock,*1 Nick A. Chappell2 and Jeffrey J. McDonnell3

1 College of Environmental Science and Forestry, State University of New York, One Forestry Drive, Syracuse, New York 13210, USA
2 Centre for Research on Environmental Systems and Statistics, Department of Environmental Sciences, Lancaster University, Lancaster, LA1 4YQ, UK
3 Department of Forest Engineering, Oregon State University, Corvallis, Oregon 97331, USA

Abstract:

Measurement uncertainty is a key hindrance to the quantification of water fluxes at all scales of investigation. Predictions of soil-water flux rely on accurate or representative measurements of hydraulic gradients and field-state hydraulic conductivity. We quantified the potential magnitude of errors associated with the parameters and variables used directly and indirectly within the Darcy–Buckingham soil-water-flux equation. These potential errors were applied to a field hydrometric data set collected from a forested hillslope in central Singapore, and their effect on flow pathway predictions was assessed. Potential errors in the hydraulic gradient calculations were small, approximately one order of magnitude less than the absolute magnitude of the hydraulic gradients. However, errors associated with field-state hydraulic conductivity derivation were very large. Borehole (Guelph permeameter) and core-based (Talsma ring permeameter) techniques were used to measure field-saturated hydraulic conductivity. Measurements using these two approaches differed by up to 3–9 orders of magnitude, with the difference becoming increasingly marked within the B horizon. The sensitivity of the shape of the predicted unsaturated hydraulic conductivity curve to ±5% moisture content error on the moisture release curve was also assessed. Applied moisture release curve error resulted in hydraulic conductivity predictions of less than ±0.2 orders of magnitude deviation from the apparent conductivity. The flow pathways derived from the borehole saturated hydraulic conductivity approach suggested a dominant near-surface flow pathway, whereas pathways calculated from the core-based measurements indicated vertical percolation to depth. Direct tracer evidence supported the latter flow pathway, although tracer velocities were approximately two orders of magnitude smaller than the Darcy predictions. We conclude that saturated hydraulic conductivity is the critical hillslope hydrological parameter, and there is an urgent need to address the issues regarding its measurement further. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS measurement uncertainty; Darcy–Buckingham equation; saturated hydraulic conductivity

INTRODUCTION

The complex processes occurring within forest soils are poorly understood. This is despite recent detailed process studies across a distribution of scales within forested mid-latitude catchments. Several key studies using combined tensiometer and hydraulic conductivity measurements have shaped our current under-
standing of hillslope hydrology, including how hollows control subsurface moisture convergence (Anderson and Burt, 1978), how macropore–matrix interactions influence mixing between old and new water (McDonnell, 1990) and how flow through weathered bedrock interacts with the soil-water system (Torres et al., 1998). A continued hindrance to the development of a conceptual model of the key runoff processes, however, lies in the uncertainty imposed by the methods and techniques used. To date, few hillslope-based studies have considered this.

Experimental uncertainty stems from two key areas: (i) that derived from the experimental approach itself, and (ii) that resulting from errors in the measurement or prediction of the parameters and variables governing soil-water flow. Studies have questioned the validity of a Darcy–Buckingham experimental approach to flow characterization, given the presence of macropores and other preferential flow pathways in many field soils (Beven and Germann, 1982; Bonell, 1998). Such features may result in turbulent water transport at velocities far greater than that within the surrounding soil matrix. Turbulent water flows within such voids cannot be described using a Darcy–Buckingham approach, and would typically result in a poor conceptualization of local water-table fluctuations, solute movement and larger scale catchment-flow dynamics. Direct measurement of the soil hydraulic properties used in flow characterization is inherently difficult and time consuming. Measurement error cannot be avoided and comes primarily from the artificial boundary conditions imposed by the measurement technique itself. Measurement precision may also be constrained by the limitations of the apparatus. The magnitude of these combined errors must, however, be quantified. Additionally, adequate spatial characterization of the soil properties is rarely achieved, and coarse averaging processes typically compromise their description across the study site.

This paper forms part of a broader study (Sherlock, 1997) designed to examine flow pathways across a distribution of hillslope units within two forested catchments in South-east Asia. Within this paper, we present the results of a hillslope-scale water and tracer flow experiment conducted in an undisturbed tropical forest catchment in central Singapore. In particular, we focus on the uncertainty issues associated with hydrometric techniques, and the implications for flow pathway prediction. The objectives of this paper are: (i) to quantify the potential parameter and variable measurement errors used in the Darcy–Buckingham equation; and (ii) to thereby determine whether relatively sound flow pathway predictions can be made from local hydrometric observations, or if such observations need to be ‘validated’ against, perhaps, direct tracer observations.

UNCERTAINTY IN SOIL-WATER FLOW ESTIMATIONS

The recent development of complex hydrological models has revealed the critical need for the incorporation of accurate field data (Bronstert, 1999; Davis et al., 1999). Yet deriving an accurate and adequately distributed data set remains highly problematic. Modelling of soil-water flux dictates that temporal and spatial distributions of field-state hydraulic conductivity, $K(h)$, and total hydraulic potential, $\psi_r$, are specified. However, studies have shown the high sensitivity of model outputs to these parameters, particularly $K(h)$ (Davis et al., 1999). Large-scale modelling efforts have been hindered because of inadequate ‘characterization of small-scale variability in near-surface soil hydraulic property information at the larger spatial and temporal scales’ (Loague and Kyriakidis, 1997). Figure 1 illustrates the breakdown of the parameters and variables typically used in water-flux calculations. Although the total potential gradient can be measured directly, deriving the $K(h)$ function for a given locality requires, in the case of this study, a multi stage process of measurement and prediction.

The moisture release curve is a critical relationship in hillslope hydrology as it conditions the shape of the $K(h)$ function, which is key to the quantitative description of flux in variably saturated media. The moisture release curve is often determined in the laboratory from pressure plate or tension table apparatus on small ‘undisturbed’ soil cores. Its derivation is tricky and error may result from: (i) poor measurement of bulk density and porosity; (ii) disturbance of the ‘undisturbed’ soil core during excavation/transport; (iii) failure of the soil core to attain moisture content equilibrium at an applied capillary potential, a common problem
Figure 1. Schematic illustrating the direct and indirect components of the Darcy–Buckingham equation.

with heavy clay cores. Furthermore, hysteresis in the moisture release curve is not usually addressed when used in subsequent analysis. The core sample also must be sufficiently large to capture the representative elementary volume (REV) of the surrounding pore-size distribution. The uncertainty associated with volumetric moisture content (θv) measurement dominates over that of capillary potential measurement (Bruce and Luxmoore, 1986). By combining tensiometry and some moisture content measurement technique such as time domain reflectometry (TDR) or neutron moderation (NM), the moisture release curve can be derived directly in the field (Torres et al., 1998). Gravimetric sampling is highly destructive, yet it is the only direct technique for measuring soil-moisture content. The TDR and NM techniques both require a calibration function, which may be erroneous for a given soil or locality. Disturbance to the natural soil system during instrumentation is a further source of uncertainty (Rothe et al., 1997).

Because the unsaturated hydraulic conductivity curve is often derived empirically from moisture release data, errors in the moisture release curve will be reflected in the resulting unsaturated hydraulic conductivity curve. Several empirical methods for unsaturated hydraulic conductivity derivation have been developed (e.g. Millington and Quirk, 1960; Brooks and Corey, 1966; Campbell, 1974; Van Genuchten, 1980), all of which produce broadly similar predictions (Chappell, 1990). To our knowledge, there are few reports of the sensitivity of these predictions to the moisture release input data. Questions still exist as to the extent to which a predicted unsaturated hydraulic conductivity curve is affected by, for example, a 5% volumetric moisture content error across the capillary potential range in the moisture release curve. Direct unsaturated hydraulic conductivity measurement is preferable, and an attempt was made by Vepraskas and Williams (1995) using the one-step outflow procedure on 345 and 6283 cm³ cores. They noted that the measured function was highly scale-dependent, and Stolte et al. (1994) demonstrated that the function was also highly technique-dependent.

In soil-water physics, most attention has been given to uncertainty associated with saturated hydraulic conductivity (Ksat) measurement, because it is a key parameter in resolving phreatic and vadose zone flux calculations (Chappell and Ternan, 1992; Davis et al., 1999). However, most field studies fail to provide an adequate spatial description of the Ksat distribution. Neilson et al. (1973), for example, reported that 576 Ksat measurements were required to characterize the conductivity distribution in a 10-ha field with a 10% accuracy at a 95% confidence level. Measurement of Ksat is time consuming as well as destructive and few studies can claim to have achieved such a comprehensive measurement distribution. Assuming an adequate sampling resolution, there remains much uncertainty as to the accuracy of the techniques currently available. Well permeameter (Talsma and Hallam, 1980; Reynolds et al., 1983; Amoozegar, 1989) and excavated core...
methods (Chappell and Ternan, 1997) are commonly used in $K_{sat}$ measurement. Yet the compatibility of the approaches is highly questionable (Bouma, 1983), as they are both highly scale-dependent and are susceptible to different types of boundary effects and associated errors.

Vepraskas and Williams (1995) and Davis et al. (1999) demonstrated the effect of core size on $K_{sat}$ measurements. In both cases, the smaller cores (347 and 264 cm$^3$, respectively) failed to capture the structural REV, with the resulting measurements failing to account for flows through macropores. Buttle and House (1997) also demonstrated that the $K_{sat}$ measured on 362 cm$^3$ cores reflected the permeability of the matrix pore-space, $K^*$, because macropores were not continuous through the cores. In their study, the core scale $K_{sat}$ measurements were up to an order of magnitude lower than bulk profile $K_{sat}$ measurements. Davis et al. (1999) showed how predictions of water-table depth and discharge were highly sensitive to $K_{sat}$ values, and that these predictions were far more accurate when large cores (11 717 cm$^3$) were used for measurement. The $K_{sat}$ scaling problem was also demonstrated by Chappell et al. (1998), who noted that $K_{sat}$ values obtained at the hillslope-scale were much greater than $K_{sat}$ values obtained at the core-scale. In all these studies, the discrepancies were attributable to poor macropore representation in the measurement volume.

Core-based $K_{sat}$ measurement approaches are theoretically more robust than well permeametry techniques, as the $K_{sat}$ measurement is based upon a direct transformation of Darcy’s Law. However, the artificial boundary conditions applied to the core on excavation may significantly affect the measured $K_{sat}$; Ritchie et al. (1972) found that cores reduced apparent $K_{sat}$, because lateral macro pore channels are blocked by the core cylinders. Other studies suggest that core approaches overestimate $K_{sat}$ in macroporous soils. Macropore channels may be continuous within an excavated core, but may not be continuous in the natural soil, with flows being naturally constrained by necks or macropore discontinuities (Sherlock et al., 1995).

Well permeametry techniques are commonly used to derive $K_{sat}$ measurements at depth. However, the greatest and most cited error associated with this approach sources from the effects of side-wall smearing of the auger hole during preparation (Wilson et al., 1989; Campbell and Fritton, 1994; Sherlock et al., 1995). Smearing can reduce measured $K_{sat}$ by several orders of magnitude, particularly in clay soils commonly found at depth. Other errors may arise from geometric errors and uncertainty of the analytical or numerical solution used to calculate $K_{sat}$. Scaling and boundary conditions associated with these different approaches ultimately result in data sets being highly incomparable for the same soil (Watts et al., 1982; Sherlock et al., 1995; Davis et al., 1999).

Tensiometry and piezometry enable the distribution and steepness of the hydraulic gradients to be calculated through a hillslope. Both techniques are simple and, assuming sufficient field maintenance, yield accurate measurements of the soil-water energy status. Where the soil is approaching saturation though, slight tensiometric errors may result in disproportionately large errors in the apparent $K(h)$ and the resulting flux calculation (Göttlein and Manderscheid, 1998).

RESEARCH SITE

The research site is located within a small 0.05 km$^2$ catchment in the Bukit Timah Nature Reserve, in central Singapore (104°E, 1°N). The catchment ranges in altitude from 100 to 164 m above sea level. Slopes vary from gentle ($<10^\circ$) on the hillslope crest zones, to moderate ($10–20^\circ$) on the mid-slopes, to steep ($>20^\circ$) in the streamside zones. The 5 × 5 m hillslope plot (Figure 2) is located on the southern slope approximately 30 m from the stream channel. The slope angle at this locality is 14·6°. The catchment is underlain by granite of the Bukit Timah Granite Formation, over which a Ferric Acrisol has developed. Catchment vegetation comprises undisturbed lowland Dipterocarp rainforest, typical of the climax vegetation across South-east Asia.

The research catchment experiences a humid tropical climate (‘tropical rainy’ under the Köppen classification system). Annual rainfall averages 2369 mm, most of which falls during short-lived convective storm events (Fook, 1992). Rainfall intensities typically range between 20 and 50 mm h$^{-1}$, although short-term (5 min) intensities can exceed 100 mm h$^{-1}$. Such rainfall inputs often cause rapid wetting of both the topsoil
and the subsoil within the catchment, resulting in highly dynamic subsurface water flow characteristics (Sherlock, 1997).

FLOW VECTOR CALCULATION

Macroscopic water flow through variably saturated porous media can be calculated from the product of the hydraulic conductivity and the total potential gradient across the medium under consideration. This forms the basis of the Darcy–Buckingham equation (Darcy, 1856; Buckingham, 1907)

\[ q = K(h) \times \frac{d(H)}{L} \]

where \( q \) is the macroscopic flux, or specific discharge, \( K(h) \) is the hydraulic conductivity as a function of capillary potential, \( h \) and \( dH/L \) is the hydraulic gradient; where \( H = h + z \), with \( z \) the elevation potential, and \( L \) is the length over which the hydraulic gradient is measured.

Where ‘nests’ of tensiometers are installed, that is at different depths through a soil profile and at different locations along a hillslope catena, two-dimensional resultant water-flow vectors can be calculated. The resultant flow magnitude (\( qR \)) and angle (\( \gamma \)) can be derived using equations presented by Harr (1977)

\[ qR = \sqrt{(qD + qV \times \sin \alpha)^2 + (qV \times \cos \alpha)^2} \]

\[ \gamma = \sin^{-1} \left( \frac{qD \times \cos \alpha}{qR} \right) \]

where \( qR \), \( qD \) and \( qV \) are the resultant, downslope and vertical macroscopic fluxes respectively, \( \alpha \) is the slope angle, and \( \gamma \) is the angle of the resultant flux, measured from vertically downwards, towards the downslope plane. Should there be a significant decrease in hydraulic conductivity down through the profile, \( \gamma \) should approach an angle parallel to the slope (i.e. \( 90^\circ - \gamma \)) above this discontinuity.
INSTRUMENTATION AND APPROACH TO UNCERTAINTY ANALYSIS

Five nests of mercury manometer tensiometers were installed at 1 m intervals down the hillslope (Figure 1). Each nest comprised five tensiometers inserted to depths of 10, 30, 50, 70, and 90 cm, which corresponded with the centre of the A, B1, B2, B3 and B4 horizon designations of the Ferric Acrisol, respectively. Capillary potential, \( h \), was measured daily over a 48-day period, with a number of discrete rainfall events being monitored more intensively. To characterize large and rapid changes in potential distributions, the sampling resolution was increased to 5–10 min over these events. The effect of error in the \( h \) and \( z \) measurements on predicted water flow is discussed and quantified. Adjacent to the tensiometer arrays, a water tracing experiment was conducted using high-flow vacuum samplers and modified resistance cells, so that the behaviour of tagged water, in this case a NaCl solution, could be compared with the calculated flow vectors. Detailed analysis of tracer behaviour is given in Sherlock et al. (1995) and Sherlock (1997).

Following the end of hillslope monitoring, a number of \( K_{\text{sat}} \) measurements were taken down the hillslope. Two techniques were used to derive \( K_{\text{sat}} \): Talsma ring permeametry and Guelph permeametry. A total of 36 Talsma ring permeametry measurements and 39 Guelph permeametry measurements were taken. The ring permeametry technique involves excavating a 7069 cm\(^3\) cylindrical core, and measuring the rate of water transfer through the saturated soil core. In contrast, Guelph permeametry adopts a borehole approach where the rate of water movement through the auger hole walls is measured. These two techniques probably represent two ends of the \( K_{\text{sat}} \) measurement spectrum, in that ring permeametry may over-estimate \( K_{\text{sat}} \) as a result of artificial boundary conditions applied to the excavated core, whereas Guelph permeametry may underestimate \( K_{\text{sat}} \) as a result of borehole smearing during preparation (Wilson et al., 1989). The \( K_{\text{sat}} \) results derived from the two techniques are therefore compared, and the effect of using the separate permeametry data sets on the resulting flux calculations (flow vectors) is quantified.

The unsaturated hydraulic conductivity curve, \( K(h) \), was derived using methods described by Millington and Quirk (1960). For this procedure, moisture release curves, \( \frac{d\theta_v}{dh} \), were derived for each horizon using pressure plate analysis on undisturbed soil cores. If the moisture release parameterization is inaccurate, the predicted \( K(h) \) function will be poor and the calculated flow vectors will be erroneous. The effect of error in the characterization of \( \theta_v/h \) on \( K(h) \) and subsequent flow predictions (\( qR, \gamma \)) are therefore discussed.

RESULTS AND MEASUREMENT ERRORS

Total potential gradients, \( \frac{dH}{L} \)

Errors in both capillary potential (\( h \)) and elevation potential (\( z \)) may arise from poor levelling, and will directly impact the total potential measurement (Figure 1). A strict levelling protocol was used during instrumentation, and the sum of these errors should not exceed 1 cm for each tensiometer. A 0.5 cm error may be introduced when determining the height difference between the tensiometer ceramic and the mercury reservoir level (i.e. \( h \pm 0.5 \)). Similarly error of 0.5 cm may be introduced when determining the elevation of each tensiometer cup above an arbitrary datum (i.e. \( z \pm 0.5 \)). Both these errors will affect values of total potential (\( H \)), the form of the equipotential net, and thus the predicted direction and magnitude of water flux. Errors of \( h \) will also cause erroneous hydraulic conductivity predictions (Göttlein and Manderscheid, 1998).

To assess the effect of errors in \( h \) and \( z \) on the total potential gradients, a PC-based program was developed that applied random errors of between \(-0.5\) and \(+0.5\) cm to \( h \) and \( z \) for each tensiometer. Given that these errors resulted from inaccurate levelling, they were held constant over the 48-day time-series. The calculated vertical and lateral potential gradients, the state-dependent hydraulic conductivity and the flow direction and magnitude were recorded through the hillslope with the applied random errors introduced to \( h \) and \( z \) for each program run. One hundred program runs were conducted on the hydrometric data set.

The effect of potential errors associated with \( h \) and \( z \) was small (Table I). Although the magnitude of the potential gradients changed over time, the potential error margin was temporally stable. Generally, vertical
Table I. Absolute range of potential vertical and horizontal hydraulic gradients derived from 100 random errors of between ±0.5 cm applied to h and z measurements of each tensiometer

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Vertical gradients</th>
<th>Depth (cm)</th>
<th>Horizontal gradients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Row A</td>
<td>Row B</td>
<td>Row C</td>
</tr>
<tr>
<td>10–30</td>
<td>0.133</td>
<td>0.145</td>
<td>0.149</td>
</tr>
<tr>
<td>30–50</td>
<td>0.157</td>
<td>0.141</td>
<td>0.146</td>
</tr>
<tr>
<td>50–70</td>
<td>0.16</td>
<td>0.139</td>
<td>0.135</td>
</tr>
<tr>
<td>70–90</td>
<td>0.139</td>
<td>0.156</td>
<td>0.126</td>
</tr>
</tbody>
</table>

gradients were between 0.6 and unity, and horizontal gradients were between 0.2 and 0.3. The variability (or range) of the measured hydraulic gradients with the applied random errors was approximately one order of magnitude lower than the absolute values of the hydraulic gradients. This indicates that possible tensiometer levelling errors did not significantly affect the magnitude of the total potential gradients, and should have had little effect on the predicted water-flow vectors.

**Saturated hydraulic conductivity,** $K_{sat}$

Recent studies suggest that the most important parameter governing soil-water behaviour is $K_{sat}$ and its spatial distribution with depth and down a hillslope (Chappell and Ternan, 1992; Davis *et al.*, 1999). Despite much attention in the literature, there remains uncertainty as to the value of such field measurements, given the potential for error. The Talsma ring and Guelph permeametry $K_{sat}$ measurements taken within this study epitomize this uncertainty. Both techniques should, theoretically, give broadly similar results. However, measurements taken within the Ferric Acrisol suggest that this is not the case (Figure 3).

Although the Guelph and ring permeametry $K_{sat}$ measurements taken within the A horizon were relatively similar, those taken within the underlying B horizon exhibited a striking contrast. For example, within the B₂ horizon the ring-based $K_{sat}$ measurements exceeded Guelph-based measurements by 1.1–3.9 orders of magnitude. Over the upper 60 cm of the soil, the Guelph-based measurements indicated that $K_{sat}$ decreased by almost four orders of magnitude, whereas the ring-based measurements indicated a decrease of only one order of magnitude. The contrast in apparent $K_{sat}$ distribution may have resulted from two key effects.

![Figure 3. Comparison of $K_{sat}$ measurements derived from Guelph and Talsma ring permeametry. All measurements in cm h⁻¹](image-url)
Firstly, the use of borehole techniques may result in \( K_{\text{sat}} \) underestimation, because they are sensitive to side-wall smearing during auger-hole preparation. Although the Guelph permeameter kit is supplied with an auger hole brush designed to negate such effects, use of the brush may still be inadequate (Wilson et al., 1989). Many of the originally conductive pores therefore may remain hydrologically inactive during measurement. This is particularly applicable to highly weathered clay-dominated soils such as the Ferric Acrisol in this study. Secondly, the ring permeameter may overestimate \( K_{\text{sat}} \) because of the artificial boundary conditions imposed at the base and around the sides of the excavated soil core. Root channels and structural voids, which continue through the length of the soil cores, will permit very rapid water flows. Overestimation of \( K_{\text{sat}} \) will result if these pathways are not naturally continuous prior to excavation of the core. To assess the possible extent of \( K_{\text{sat}} \) overestimation, permeametry measurements were repeated on several of the excavated cores following clay sealing of all the pores \( >2 \text{ mm} \) in diameter at the base of the core. It was envisaged that this measurement would give a \( K_{\text{sat}} \) value approaching that of the matrix pore-space, \( K^{*} \). Seven such tests suggested that the \( K^{*} \) of the cores was between 0.5 and 2.1 orders of magnitude lower than the measured \( K_{\text{sat}} \), and may be of greater use in soil-water flow prediction.

**Unsaturated hydraulic conductivity, \( K(h) \)**

The pore-size distribution of the soil determines the shape of the moisture release curve. This in turn conditions the hydraulic conductivity curve, \( K(h) \). Using the Rawls and Brakensiek (1989) technique for moisture release prediction, Sherlock (1997) indicated that volumetric moisture content errors of up to 5% may be incurred if approximate estimates of sand and clay content are used as input parameters to the model. The effect of a 5% volumetric moisture content error on the Millington and Quirk (1960) prediction after matching to horizon-specific ring permeametry \( K_{\text{sat}} \) measurements is illustrated in Figure 4. Differences between the \( K(h) \) curves derived from the Rawls and Brakensiek (1989) modelled moisture release curves, and the \( K(h) \) curves derived from the modelled moisture release curves with an applied 5% moisture content error are small, particularly when the soil approaches saturation. Within the A horizon differences are below 0.1 order of magnitude over the capillary potential range 0 to \(-600 \text{ cm } \text{H}_2\text{O}\). In the B\(_1\) horizon, the differences of less than 0.1 order of magnitude extend over the whole capillary potential range illustrated. Greater potential for error is evident within the underlying horizons. Within the B\(_2\) horizon, the differences ranged between 0.1 and 0.2 order of magnitude, and in the B\(_3\) horizon, the differences approached 0.5 order of magnitude at very low capillary potentials. However, tensiometric measurements indicate that soil-water status in the B\(_3\) horizon rarely fell below \(-20 \text{ cm } \text{H}_2\text{O}\) over the 48-day monitoring period (Sherlock, 1997). As soil-water flux is very small at low capillary potentials, the absolute error in the magnitude and direction of the flux prediction is not considered significant.

Measurements of capillary potential are used to define the magnitude of the hydraulic conductivity through the hillslope (Figure 1). In addition to the total potential gradients, capillary potential measurement errors of \( \pm 0.5 \text{ cm} \) will also affect the apparent hydraulic conductivity values used in subsequent Darcy–Buckingham flux calculations. Using the same analytical approach conducted on the total potential gradient data set, the effect of 100 random errors of between \( \pm 0.5 \text{ cm} \) applied to the capillary potential data set on the apparent hydraulic conductivity distribution was assessed. Figure 5 illustrates the magnitude of the range of computed hydraulic conductivity values, plotted against measured capillary potential. Data from all tensiometer arrays are plotted, and demonstrate that for all depths, the magnitude of the hydraulic conductivity range is generally less than 0.05 orders of magnitude. However, where the soil becomes near-saturated \((-5 < h < 0 \text{ cm } \text{H}_2\text{O})\), the potential error increases to between 0.2 and 0.35 order of magnitude. This results from the rapid change in hydraulic conductivity with capillary potential over this moisture content range. The error range of the calculated hydraulic conductivity values is zero above \( +0.5 \text{ cm } \text{H}_2\text{O} \) capillary potential, as the state-dependent hydraulic conductivity equals \( K_{\text{sat}} \) (which remains constant at all positive pressure heads).
EFFECT ON RESULTANT FLOW VECTOR CALCULATIONS

Capillary potential measurement error affects calculated flux in two direct ways: through the total potential gradient and through the $K(h)$ function (Figure 1). Given that the total potential gradients were only slightly affected by possible tensiometer levelling errors, the effect on calculated flow vectors should be small. However, we showed that erroneous capillary potential measurements resulted in poor predictions of hydraulic conductivity when the soil neared saturation. We therefore examined the effect of the $\pm 0.5 \text{ cm H}_2\text{O}$ capillary potential error on the resultant flow vectors. The error range was determined using a PC-based program, which first applied a random error of between $\pm 0.5 \text{ cm H}_2\text{O}$ to each of the 25 measured capillary potential time-series data sets, and secondly, computed the flux magnitude based upon the capillary potential measurements with each of the applied errors. The program was run 100 times, and the range of flux between each horizon and tensiometer array was derived for each time of capillary potential measurement. These error ranges were plotted against the measured capillary potential from the shallowest and most upslope tensiometer associated with each flux calculation for each capillary potential measurement time (Figure 6a–d). The spread is small at all depths, with fluxes typically ranging over less than 0.15 order of magnitude. However, the magnitude of the flux range increases markedly to between 0.35 and 0.4 order of magnitude.  

Figure 4. Effect on $K(h)$ curve of a $\pm 5\%$ volumetric moisture content error in the moisture release curve. $K_{sat}$ curves matched to ring permeameter $K_{sat}$ measurements are used to illustrate the effect through the Ferric Acrisol profile.
Figure 4. Error range of depth-specific hydraulic conductivity resulting from \(+0.5\) cm H$_2$O capillary potential error

Figure 5. Error range of depth-specific hydraulic conductivity resulting from \(\pm 0.5\) cm H$_2$O capillary potential error

magnitude as the soil approaches saturation. This reflects the sensitivity of the \(K(h)\) function to measured \(h\) under near-saturated conditions. In terms of absolute velocities, the uncertainty associated with capillary potential measurement could result in calculated velocities exceeding the true velocities by up to 100%, and vice versa.

Examination of the hydrometric data during a high-magnitude storm clarifies this point. The storm commenced at 12:33 h on 23 April 1993, and delivered 112 mm rainfall over a 3-75 h period. Figure 7 illustrates the flow vectors between the A and B$_1$ horizons in the uppermost portion of the instrumented hillslope over the course of the storm event. As the magnitude of the flux increases in response to soil wetting, the sensitivity of the flux calculation to capillary potential error increases. For example, at the height of the storm, flow magnitude ranged between 50 and 80 cm h$^{-1}$. The direction of flow was not affected significantly by the introduced capillary potential errors, with the range of angles not exceeding 10° through the entire instrumented hillslope over the course of the monitoring period.

State-dependent hydraulic conductivity is the constant of proportionality used within the Darcian calculations. Although the predicted shape of the \(K_{rel}\) function was not affected significantly by slight errors in the moisture release curve, \(K(h)\) will be affected directly by the local \(K_{sat}\) measurement to which the \(K_{rel}\) function is matched. One way of examining the effect of \(K_{sat}\) measurement error is to assume that the Talsma ring and Guelph permeametry derived measurements represent the upper and lower \(K_{sat}\) measurement limits for a soil. Table II summarizes the difference between averaged Talsma ring and Guelph-based flow vectors within the hillslope. Over the experimental period, averaged ring-based velocities were far greater than Guelph-based velocities. This was particularly marked within the argic B horizon, where the difference between the ring and Guelph-based velocities exceeded 2.5 orders of magnitude. Applying the contrasting \(K_{sat}\) data sets to the flow calculations also produced very different predictions of hillslope flow pathways. Ring permeametry-based calculations indicated a dominant vertical flow component, whereas the Guelph-based flows identified significant near-surface lateral flow. Greater hydraulic discontinuities in the vertical plane, as suggested by the Guelph permeameter (Figure 4), invariably result in increased flow deflection downslope (Zavlavsky and Sinai, 1981). Within this forested Ferric Acrisol, water-flow vectors derived using the contrasting \(K_{sat}\) profiles result in a very different conceptualization of water-flow behaviour. The flow
Figure 5. Potential flux range through the soil profile versus measured capillary potential from the shallowest and most upslope tensiometer associated with each flux calculation.

Figure 6. Variability of the magnitude and direction of flow vectors between the A and B horizon horizons with introduced errors of ±0.5 cm to h and z measurements.
Table II. Averaged ring permeameter and Guelph permeameter-derived average linear pore-water velocities \((v_r, v_g)\) and flow directions \((\gamma)\) of spatially averaged flow vectors compared with tracer flow linear velocities \((v_t)\) and directions. All velocities in cm h\(^{-1}\)

<table>
<thead>
<tr>
<th>Depth (horizons) (cm)</th>
<th>Ring permeameter-based vectors</th>
<th>Guelph permeameter-based vectors</th>
<th>Difference</th>
<th>Tracer flow characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(v_t)</td>
<td>(v_g)</td>
<td>((\log v_t) - (\log v_g))</td>
<td>(v_t)</td>
</tr>
<tr>
<td>Linear velocity</td>
<td></td>
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<tr>
<td>10–30 (A – B(_1))</td>
<td>15.5</td>
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<td>30–50 (B(_1) – B(_2))</td>
<td>4.1</td>
<td>0.2</td>
<td>1.35</td>
<td>0.035</td>
</tr>
<tr>
<td>50–70 (B(_2) – B(_3))</td>
<td>3.8</td>
<td>0.01</td>
<td>2.53</td>
<td>0.0069</td>
</tr>
<tr>
<td>70–90 (B(_3) – B(_4))</td>
<td>2.6</td>
<td>0.006</td>
<td>2.65</td>
<td>0.056</td>
</tr>
<tr>
<td>Flow angle (degrees)</td>
<td>(\gamma_t)</td>
<td>(\gamma_g)</td>
<td>(\gamma_g - \gamma_t)</td>
<td>Tracer direction, (\gamma_t)</td>
</tr>
<tr>
<td>10–30 (A – B(_1))</td>
<td>28.1</td>
<td>50.6</td>
<td>22.5</td>
<td>Vertical</td>
</tr>
<tr>
<td>30–50 (B(_1) – B(_2))</td>
<td>26.9</td>
<td>66.3</td>
<td>39.4</td>
<td>Vertical</td>
</tr>
<tr>
<td>50–70 (B(_2) – B(_3))</td>
<td>19.1</td>
<td>23.4</td>
<td>4.3</td>
<td>Vertical</td>
</tr>
<tr>
<td>70–90 (B(_3) – B(_4))</td>
<td>13.9</td>
<td>13.9</td>
<td>0</td>
<td>Vertical</td>
</tr>
</tbody>
</table>

predictions need to be validated against some other hydrological approach, in this case a sodium chloride (NaCl) tracing experiment.

FLOW VECTOR VALIDATION USING DIRECT TRACER OBSERVATIONS

A NaCl tracing experiment was conducted adjacent to the hydrometric (tensiometry/permeametry) experimental area (Figure 1). The tracer was injected as a line source along the upslope margin of the instrumented area. The downslope and vertical migration of the tracer was monitored using a combination of modified Colman resistance cells (to sample mobile/macropore tracer flows) and suction lysimeters (to sample immobile/matrix tracer flows). For the purpose of validating the flow-vector predictions, it is useful to make a comparison between the Darcy-based water-flow predictions and the water flows directly observed using the tracing approach. The NaCl breakthrough curves at sampling points vertically below and 1 m downslope of the line source injection are illustrated in Figure 7.

The tracing experiment indicated that water flux within this soil was vertical through the upper 89 cm of the profile. Downslope migration of the NaCl tracer was not observed by either the suction lysimeters or the resistance cells (Figure 8; Sherlock, 1997). This compares well with the predominantly vertical Talsma ring permeametry-based flow directions (Figure 7; Table II). However, the Guelph permeametry-based predictions indicated a dominant lateral flow-pathway through both the A and upper B horizons, suggesting that the combination of techniques used in this predictive approach was inadequate. Given the discussion above, most of the error in this approach probably resulted from inaccurate measurements of \(K_{sat}\).

For the purpose of tracer and Darcy-based flow velocity comparison, specific flux \((q)\) calculated using the Darcy–Buckingham equation was converted to linear pore-water velocity using:

\[
v_r, v_g = \frac{q}{\eta}
\]  

where \(v_r\) and \(v_g\) are the ring and Guelph permeametry-based linear pore-water velocities, and \(\eta\) is the porosity. The average linear velocity of the tracer, \(v_t\), was calculated by dividing the elapsed time between tracer injection and measurement of the centre of the tracer mass (determined from the centroid of the breakthrough curve), by the linear distance between the injection source and the sampling point. Neither the ring nor the
Guelph-based flow calculations were successful in predicting the average velocity of the tracer mass, $v_t$. The ring permeametry-derived pore-water velocities, $v_r$, exceeded the mean velocity of the tracer by over two orders of magnitude (Table II). This was consistent through the soil profile. Given that the flow-vector predictions are most sensitive to $K_{sat}$ error, this suggests that the ring technique overestimates the real $K_{sat}$ of this Ferric Acrisol. Guelph permeametry-based flow velocities, $v_p$, averaged one order of magnitude lower than the tracer mass velocity, indicating an underestimation of the field $K_{sat}$. These general observations support the argument that both Talsma ring and Guelph permeametry techniques are potentially erroneous, and that the real $K_{sat}$ of a clayey, macroporous soil would probably fall between these two ‘extremes’.

CONCLUSIONS

Predictions of water flow and subsequent conceptualization of the hillslope hydrological processes are highly sensitive to measurement error and the uncertainties imposed by the techniques themselves. By applying a coarse sensitivity analysis to the Darcy–Buckingham equation used for predicting water-flow vectors in a
Ferric Acrisol, we exposed several weaknesses that must be addressed when using this type of predictive approach. Mathematically, the equation is equally sensitive to the magnitude of the hydraulic gradient \((dH/L)\) and the hydraulic conductivity \((K(h))\). However, the magnitude of the potential error in the hydraulic gradient is far less. The effect of small errors in capillary and elevation potential on the total potential gradient magnitude was shown to be minor, and thus did not affect the resulting flow-vector calculations. The sensitivity of Millington and Quirk (1960) analysis to the moisture release curve also had negligible effect on the flow calculations. The potential for error in the magnitude of hydraulic conductivity was, however, very large. Firstly, when the soil was near-saturated (over the \(-5 < h < 0\) cm H\(_2\)O range), the state-dependent hydraulic conductivity became very sensitive to capillary potential. Secondly, the magnitude of measured \(K_{\text{sat}}\) varied by several orders of magnitude, depending upon the type of approach used in measurement, and resulted in very different hydrological flow pathway predictions.

The results of this work hold important implications for other studies that have used measurements of hydraulic conductivity and total potential to estimate hillslope flow behaviour. The results of widely cited hillslope papers by Weyman (1973), Harr (1977), Anderson and Burt (1978) and McDonnell (1990) that have shaped our current understanding of hillslope flow processes must be assessed in light of experimental uncertainty. Given our findings, the flow direction and pathways suggested in previous studies are probably uncertain, affecting their conceptualization of hillslope flow behaviour.

Finally, site or study comparison is greatly hindered by the different (particularly \(K_{\text{sat}}\)) techniques used, and one may rightly question whether different hydrological behaviours are real, or whether contrasts are purely a function of the different techniques used. We stress the difficulty in deriving representative hydrological parameters, with particular reference to \(K_{\text{sat}}\). Although it is a fundamental parameter in soil-water physics, measurement uncertainty can be significant; yet this uncertainty must be refined if we are to make sound, acceptable predictions of soil-water flow.

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REFERENCES


