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Identification of the dominant runoff pathways from data-based mechanistic modelling of nested catchments in temperate UK

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SUMMARY

Understanding hydrological flow pathways is important for modelling stream response, in order to address a range of environmental problems such as flood prediction, prediction of chemical loads and identification of contaminant pathways for subsequent remediation. This paper describes the use of parametrically efficient, low order models to identify the dominant modes of stream response for catchments within the Upper Eden, UK. A first order linear model adequately identified the dominant mode in all but one of the sub-catchments. A consistent pattern of time constants and pure time delays between catchments was observed over different periods of data. In the nested catchments, time constants increased as the catchment size increased from 1.1 km^2 at Gais Gill (2–7 h) to 69.4 km^2 at Kirkby Stephen (5–10 h) to 223.4 km^2 at Great Musgrave (7–16 h) to 616.4 km^2 at Temple Sowerby (11–22 h), but Blind Beck (a small catchment 8.8 km^2, time constants 11–21 h) had time constants most similar to Temple Sowerby. This was attributed to a combination of the storage role of permeable rock strata, where present, and the effect of scale on sub-surface and channel routing. A first order model could not be identified for the 1.0 km^2 Low Hall catchment, which comprises permeable sandstone overlain by Quaternary sediments. A second-order model of Low Hall stream showed a higher proportion of water taking a slower pathway (76% via a slow pathway; time constant 252 h) than a model with the same structure for the 8.8 km^2 Blind Beck (46% via slow pathway; time constant 60 h), where only 38% of the basin was underlain by the same permeable sandstone. This highlights the need to quantify the role of deep pathways through permeable rock, where present, in addition to the effect of catchment size on response times.

1. Introduction

Water may travel along many different flow pathways through a catchment, including fast pathways, such as overland flow, shallow sub-surface flow, pipe flow or drain flow, and slower pathways such as sub-surface flow through the deeper strata of regolith and rock. Knowledge of these pathways is particularly important in water quality studies as different pathways may have different chemical signatures. The way in which these pathways are represented in hydrological modelling depends on the type and structure of the model.

There exists a range of hydrological models of varying complexity, classified into various types by Wheater et al. (1993), for a range of scales from plot scale (<1 ha) to large catchment scale (>1000 km^2), and the choice of an appropriate model depends on both the modelling task and the input information available. If the model is assessed on successful simulation of the stream hydrograph alone, then there may be many good models with different parameter sets which give an equally acceptable fit to the calibration data; this is the concept of 'equifinality' (Beven, 1996, 2006; Beven and Freer, 2001). However, because of the lack of data on internal variables, some of the models may fit for the wrong reasons and if predictive models are to be used with confidence for conditions beyond our current experience (e.g. for climate change assessment) then it is necessary to get the right answer for the right reasons (Kirchner, 2006; Kirchner et al., 1996; Seibert and McDonnell, 2002). McDonnell (2003) suggests that observed non-linear responses in catchments, due to threshold behaviour, hysteresis and storage effects, challenge conventional model structure and some of its assumptions.

Uhlenbrook et al. (2003) recommend the construction of models that capture the dominant modes of a system, with as few tuneable parameters as possible, as an increase in the number of parameters also increases the uncertainty of the model. Transfer function models, whose structure and parameters are determined by the information in the data are considered to be among the most parsimonious for investigating rainfall–runoff (McGuire and McDonnell, 2006; Young, 2003). This is the approach used in data-based mechanistic (DBM) modelling.

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The objective of this study was to use DBM modelling to investigate the dominant pathways of the water expressed in the stream response of the Upper Eden catchment and to see if and how these changed with catchment scale or underlying geology.

2. Materials and methods

2.1. Catchment descriptions

The study area was the Upper Eden catchment, situated in north-west England, UK. The River Eden is 80 miles long and rises in the limestone on the border of Cumbria and Yorkshire and then runs north through the towns of Kirkby Stephen and Appleby, and onto Carlisle city where it discharges into the Solway Firth. The Upper Eden is the part of the catchment above the flow gauging station at Temple Sowerby (616 km^2; Lat. 54.65°N, Long. 2.61°W). Elevations range from around 90 m above sea level in the valley bottom, to nearly 800 m in the Pennines on the eastern side. The Eden Valley is made up of Permian and Triassic sandstones overlying rocks of the Carboniferous Series. It is separated from the Carboniferous rocks of the Pennines by the Pennine fault (Taylor et al., 1971; Taylor, 2003). Both the Penrith sandstone and the St Bees sandstone in the valley bottom are defined as major aquifers (Allen et al., 1997). In contrast, the valley sides are mainly Carboniferous limestone and limestone layers. The headwater catchment of Gais Gill in the south-west of the catchment consists of an impermeable greywacke. The valley floor is covered by a variable thickness of Quaternary drift made up of poorly sorted mixtures of clays, silts, sands and gravels interspersed with alluvial deposits. Coarse river terrace deposits cover a large proportion of the small catchment of Low Hall. The steeper sides of the valley mostly have no covering of drift. Rainfall totals in the Upper Eden vary from less than 650 mm per year in the valley bottom to over 2000 mm per year on the fells and have been linearly related to elevation (Walsh, 2004). The Upper Eden is primarily used for livestock farming, with sheep on the upland areas and beef or dairy cattle on the more fertile lowland areas.

The Eden is one of four UK catchments in the Catchment Hydrology and Sustainable Management (CHASM) research programme. CHASM is a UK framework for catchment research to address issues such as flooding, water quality, ecology and land use over a range of scales from plot scale to basin scale (Quinn et al., 2000). As a result, the Upper Eden is relatively densely instrumented and provides hydrometric data for a database maintained by Newcastle University.

The sub-catchments modelled ranged in size from Temple Sowerby (616 km^2) to Gais Gill (1.2 km^2) and Low Hall (1.0 km^2) (Fig. 1). The location of the gauging station for each sub-catchment and the percentages of limestone, sandstone and drift deposits are given in Table 1. The larger sub-catchments are gauged by the Environment Agency (EA).

2.2. Data-based mechanistic (DBM) models

DBM philosophy makes no prior assumptions about the process-descriptions of model structure but allows the data to determine the structure, which is then interpreted in terms of physical processes. The modeller starts with time-series data and fits a range of models such as transfer function models. An optimal model and associated parameters is identified using statistical measures, but only accepted if it has a plausible physical explanation (Young, 1998, 2003; Young and Beven, 1994; Young et al., 2004). Parameter uncertainty is identified as part of the model identification process and is constrained by keeping the number of parameters to the minimum required to explain the data, which then allows meaningful interpretation of parameter differences (e.g., between several catchments).

Transfer function models are the core of many DBM models. A transfer function is a linear relation between an input and an output via one or more stores. The stores deplete exponentially, which is a constraint on DBM models that use transfer functions. A general, discrete-time, single-input, single-output transfer function model can be written as:

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})} u(k - \delta) \tag{1}$$

where $u(k)$ represents the input at the kth sampling time, $y(k)$ represents the output at the kth sampling time and δ is a pure time delay. The polynomials A and B are defined as:

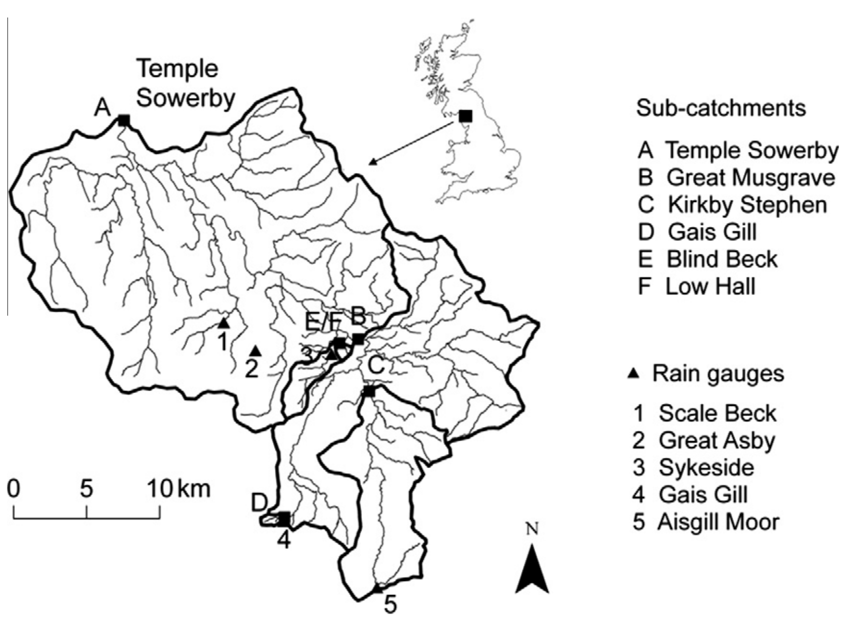

Fig. 1. The Upper Eden catchment within the UK, showing sub-catchments and location of gauging stations and rain gauges. The river gauging stations are shown with filled squares in the map, while the rain gauges are shown with filled triangles. The 1.0 km^2 catchment area of Low Hall stream is too small to be represented in this map; its gauging station is adjacent to that of the 8.8 km^2 Blind Beck gauging station, so both are shown with the same symbol.

Table 1
Summary of sub-catchments and gauging stations used in modelling. L = Limestone, SST = Penrith/St Bees Sandstone (see Allen et al., 1997; British Geological Survey, 1997). OS is the UK Ordnance Survey.

Sub-catchment	Area (km^2)	Geology	Gauge location OS Grid ref	Gauge maintained by
Temple Sowerby	616.4	69% L 28% SST	NY605283	EA
Great Musgrave	223.4	83% L 13% SST	NY764131	EA
Kirkby Stephen	69.4	91% L 6% SST	NY773097	EA
Gais Gill	1.1	100% Greywacke 0% SST	NY714011	CHASM
Blind Beck	8.8	62% L 38% SST (regolith = 61% till, 17% river terrace)	NY753131	CHASM
Low Hall stream	1.0	100% SST (regolith = 40% till, 60% river terrace)	NY753130	This project

$$A(z^{-1} = 1 + a_1 z^{-1}) + a_2 z^{-2} + \cdots + a_n z^{-n} \tag{2}$$

and

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m} \tag{3}$$

where z^{-i} is the backwards shift operator, i.e. $z^{-i}y(k) = y(k-i)$.

The characteristics of the model depend on the polynomials A and B, in particular on their roots. The structure of a simple transfer function model is sometimes denoted by the triad $[n, m, \delta]$ or the pair $[n, m]$ if the pure time delay is excluded (Young and Beven, 1994).

A simple first order transfer function model with structure $[1, 1, \delta]$ can be written using the notation of Eq. (1) as

$$y(k) = \frac{b}{1 + az^{-1}} u(k - \delta) \tag{4}$$

This system will have a characteristic time constant, TC, in units of the model time step, and a steady state gain, SSG, defined by

$$TC = \frac{-\Delta t}{\ln(-a)}; \quad SSG = \frac{b}{1 + a} \tag{5}$$

where Δt is the time step of the model, determined by the sampling resolution of the time-series data (or an integer integral of the sampling resolution), and a and b are the parameters defined in Eq. (4).

When this model is applied to the rainfall–runoff response of a catchment, typically the input to the model is a rainfall time series (e.g., mm/h) and the output is a river runoff time series (e.g., mm/h) sampled at the same time interval (Young, 2003). The conceptual interpretation of the linear store expressed by Eq. (4) is the depletion of a single uniform store; normally interpreted as the drainage from a sub-surface moisture store into the river. The time constant is a measure of how fast the system responds, measured by the speed of the recession. There may be a delay (δ) before any response at all is seen in the output (a pure time delay). This does not necessarily mean that the response seen in each stream hydrograph is the same water that fell in the rain-event immediately preceding each hydrograph, as studies have indicated that there can be a 'groundwater ridging' phenomenon with the stream response being made up of 'old' water that has been in the ground for some time (Cloke et al., 2006; Pearce et al., 1986; Sklash and Farvolden, 1979; Sklash et al., 1986).

If the rainfall used as model input is representative of the catchment average rainfall, then the SSG is the ratio of the catchment average rainfall, integrated over the modelled period, to the integrated river runoff. Thus, if a SSG of 0.5 was obtained, the runoff total is equivalent to half of the rainfall total. The difference comprises evapotranspiration outputs or groundwater exchanges across catchment divides or changes in sub-surface storage, or some combination of all these. In catchments without groundwater exchanges across divides and modelling periods normally in excess of 1 year, the difference between the rainfall input and river discharge output (i.e. P–Q) equates largely to the evapotranspiration output for the catchment.

For some catchments within this study, rainfall input was measured at a maximum of two rain gauges. As a consequence, the catchment average rainfall could not be derived to give even an approximate estimate of the annual evapotranspiration (from an integral of P–Q). Even with data from a detailed rain gauge network, uncertainties remain with evapotranspiration estimates from P–Q. Thus, in this study, the SSG is not interpreted. However, within temperate catchments of the Eden where the rainfall regime is predominantly frontal, the rain-event hyetographs for all rain gauges within a catchment are expected to have approximately the same linearly-scaled shape (e.g. Mayes et al., 2006).

Another commonly identified model in hydrology has two linear stores in parallel, which can be interpreted as a parallel, fast response pathway with a short time constant and a slower pathway with a longer time constant. Provided the pure time delay is the same on both pathways, two first order transfer functions can be added, resulting in a second order [2, 2] model, which has real roots. If such a model is identified, the time constants and relative proportions of water taking each pathway can be determined from a partial fraction decomposition of the second order transfer function (e.g. Beven, 2001, p. 109).

Within this study, model structure and parameters are identified using the Simplified Refined Instrumental Variable algorithm, an iterative process giving a set of unbiased parameters which optimises the model whilst also estimating the uncertainty on the model parameters. This is carried out using the CAPTAIN toolbox which runs within Matlab® (Taylor et al., 2007). Models are evaluated according to the coefficient of determination, R_T^2, and the Young Identification Criterion (YIC) (Young, 1984), an objective statistical measure which combines how well the model fits the data and a measure of the over-parameterisation. R_T^2 is defined as:

$$R_T^2 = 1 - \frac{\hat{\sigma}^2}{\sigma_y^2} \tag{6}$$

where $\hat{\sigma}^2$ is the variance of the model residuals and σ_y^2 is the variance of the observations (output) y.

YIC is defined as:

$$\text{YIC} = \log_e \frac{\hat{\sigma}^2}{\sigma_y^2} + \log_e\{\text{NEVN}\} \tag{7}$$

where NEVN is the normalised error variance norm defined as:

$$\text{NEVN} = \frac{1}{np} \sum_{i=1}^{np} \frac{\hat{\sigma}^2 \hat{P}_{ii}}{\hat{a}_i^2} \tag{8}$$

np is the number of parameters estimated, $\hat{P}_{ii}$ is the ith diagonal in the parameter covariance matrix, $\hat{a}_i^2$ is the square of the ith parameter. The first term in Eq. (7) is based on the coefficient of determi-

Table 2
Summary of rain gauges used in modelling.

Location	Grid reference	Elevation (m)	Maintained by
Aisgill Moor	SD778963	360	EA
Scalebeck	NY673144	183	EA
Gais Gill	NY714009	390	CHASM
Great Asby	NY695125	250	CHASM
Sykeside	NY747122	180	CHASM

nation and is a measure of how well the model explains the data (the smaller the model residuals, the more negative this term becomes). The second term represents the impact of uncertainty in the parameter values, which is correlated with model complexity (or over-parameterisation); generally, a higher order model will capture more of the dynamics of the system, but with higher uncertainty in the parameter estimates. In that case the second term in YIC will dominate. Thus YIC is a compromise between the fit of the model and model complexity.

2.3. Data preparation

High temporal resolution data of rainfall and flow (i.e., 15 min resolution) were required in order to capture the storm-event dynamics which are observable at the scale of the smallest modelled sub-catchment (~1 km^2). Rainfall data from several weather stations were required in order to choose the most appropriate series for the scale of the catchment being modelled. Rainfall data from Scalebeck and Aisgill Moor were obtained from the Environment Agency as 15 min time series. Rainfall data from weather stations at Sykeside (NY747122), Great Asby (NY695125) and Gais Gill (NY714009) were supplied by CHASM as tipping bucket times for the period January 2007–February 2009. The tip times were integrated over 15 min intervals to create a time series at 15 min resolution. A summary of the rain gauges is given in Table 2.

Flow data were required for catchments of different scale and on different geologies. Water level data for the gauging sites at Kirkby Stephen, Great Musgrave and Temple Sowerby (see Fig. 1.) were supplied by the Environment Agency as time series at 15 min resolution. Levels were converted to discharge using the ratings published by the Environment Agency (http://www.environment-agency.gov.uk/hiflows/91727.aspx). The flow data for Blind Beck and Gais Gill were obtained from CHASM as time series at 15 min resolution. Discharge values were based on stage-discharge relationships built up by Newcastle University over several years. The maximum rated flow in Blind Beck was 1.32 m^3 s^{-1} (exceeded by 4% of flows during January 2006–December 2008. For the smallest sub-catchment of Low Hall, within the catchment of Blind Beck (entirely on the sandstone), water level data at 15 min resolution and water level to discharge rating were collected by the lead author during the period December 2007–December 2008. The maximum rated flow in Low Hall stream was 0.12 m^3 s^{-1} (exceeded by 4% of flows in the period December 2007–December 2008). The river discharges (m^3 s^{-1} per 15 min or 1 h time steps) were converted to runoff (i.e., mm/h) by dividing by the respective catchment areas (Table 1).

2.4. Time periods modelled

Rainfall–runoff was modelled over both long periods (up to 4 years where data were available) and short periods (minimum 8 days) to see if trends in time constants between catchments were consistent. Data were modelled with an hourly time step, as this gave sufficient sampling of the dynamics in each catchment. For all the sub-catchments in Fig. 1, rainfall–runoff was modelled using a range of linear transfer function models from first to third order,

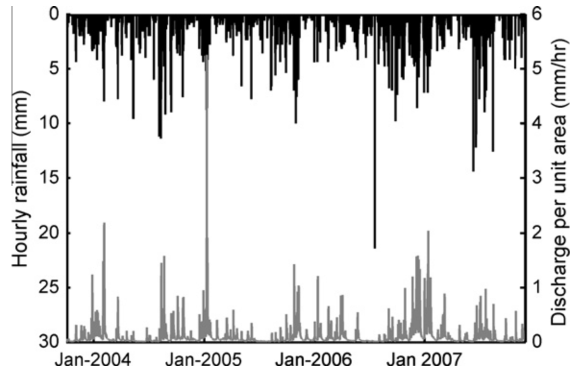

Fig. 2. Hourly rainfall at Scale Beck and hourly discharge per unit area at Temple Sowerby, 01 December 2003–30 November 2007. Water levels and rainfall were supplied at 15 min resolution by the Environment Agency.

with a time delay up to 8 h (8 time steps), i.e. with a structure [1 1 0] to [3 3 8].

3. Results and discussion

3.1. Time constants and pure time delays for the Upper Eden

The hourly rainfall at Scale Beck and hourly discharge per unit area at Temple Sowerby is shown in Fig. 2 for the 4 year period 1 December 2003–30 November 2007. The highest runoff rates are generally recorded in the winter months, and these usually correspond with heavy rainfall. In the winter months, this rainfall is generally from frontal rain systems which give a relatively smooth trend of rain across the whole catchment (Walsh, 2004). However, Fig. 2 also shows that the most intensive rainfall events in the summer months are not always followed by a corresponding rise in the runoff. This is probably due to the greater prevalence of convective rainfall in the summer, which falls intensively over small areas, thus making a single rain gauge much less representative of the larger catchment. An extensive network of rain gauges is therefore important (Wilkinson, 2009), particularly for modelling focused only on convective storms.

Fig. 2 also shows the frequently observed and well documented non-linearity of the rainfall to flow (e.g. Jakeman et al., 1990; Whitehead et al., 1979; Young and Beven, 1994), where the rain falling in summer results in a much smaller stream response than the same amount of rain in winter when the catchment is much wetter. As the purely linear models identified mostly had a high R_T^2, the additional parameterisation of the non-linearity was not added to aid identifiability of the time constants of the linear transfer functions. For the catchments of Temple Sowerby, Great Musgrave, Kirkby Stephen, Gais Gill and Blind Beck the data indicated that a first order model with a pure time delay was, for almost all periods modelled, the simplest model that best explained the data (most negative YIC). Table 3 summarises the model results over the different time periods chosen. The rainfall used in each case, the model structure identified, the model fit (coefficient of determination and YIC) and time constant are also shown. In all these catchments, the time constants are several times larger than the sampling interval, therefore the sampling interval does not impact on the accuracy of the time constants derived from discrete time modelling (Littlewood and Croke, 2008; Littlewood et al., 2010). Where rainfall–runoff has been modelled for a year or more, the time constants are longer than for the same catchment modelled for several months. The model fit is also less good for the longer periods. This is because the dynamics within the catchment may change between the seasons and time constants for the

Table 3
Summary of modelling results comparing time constants for different sub-catchments over different periods. R_T^2 is the coefficient of determination, YIC is the Young Information Criterion (see Young, 1984).

Time period	Catchment	Rain gauge	Model structure	Time delay (h)	R_T^2	YIC	Time constant (h)
4 years: October 2003–November 2007	Temple Sowerby	Scale Beck	[1 1 5]	5	0.68	−7.2	21.7
	Great Musgrave	Scale Beck	[1 1 3]	3	0.64	−7.3	15.8
	Kirkby Stephen	Aisgill Moor	[1 1 2]	2	0.68	−11.1	9.3
16 months: November 2006–April 2008	Blind Beck	Sykeside	[1 1 0]	1	0.67	−5.9	20.7
4 months: December 2006–April 2007	Temple Sowerby	Scale Beck	[1 1 5]	5	0.91	−11.4	16.3
	Great Musgrave	Scale Beck	[1 1 3]	3	0.85	−10.3	12.4
	Kirkby Stephen	Aisgill Moor	[1 1 2]	2	0.84	−10.1	6.5
	Gais Gill	Gais Gill	[1 1 0]	0	0.83	−9.9	6.5
	Blind Beck	Sykeside	[1 1 1]	1	0.84	−6.3	20.6
8 days: 30 January 2004–7 February 2004	Temple Sowerby	Scale Beck	[1 1 5]	5	0.94	−10.1	11.3
	Great Musgrave	Scale Beck	[1 1 2]	2	0.88	−8.7	7.6
	Kirkby Stephen	Scale Beck	[1 1 2]	2	0.85	−8.1	5.2
	Gais Gill	Scale Beck	[1 1 0]	0	0.66	−5.7	2.5
	Blind Beck	Scale Beck	[1 1 1]	1	0.86	−8.5	10.9

summer may be different. The time constant derived from several years of data therefore represents an average for the changing dynamics of the system through the seasons.

The relative difference between the time constants for different catchments is, however, consistent regardless of the length of the period modelled. There is a trend of increasing time constant from Gais Gill to Kirkby Stephen to Great Musgrave to Temple Sowerby. This is not just related to an increase in catchment size as time constants in Blind Beck (8.8 km^2) are similar to Temple Sowerby (616 km^2). The fastest response of all is seen in the Gais Gill catchment, a small, steep, upland catchment on an impermeable greywacke geology (British Geological Survey, 1997). As the catchment size increases from Kirkby Stephen to Temple Sowerby, so does the proportion of the catchment on permeable sandstone (see Table 1) with the potential for deeper pathways and greater storage.

The pure time delay also increases as the catchment size increases, with no time delay at Gais Gill, 2 h at Kirkby Stephen, increasing to 5 h at Temple Sowerby. Time constants for the Blind Beck catchment, a small catchment of only 8.8 km^2, are much more similar to those of the Temple Sowerby catchment than the catchments with a smaller proportion of permeable sandstone between 1 and 69 km^2 in area. Table 1 also shows that the geology of the Blind Beck catchment is most similar to Temple Sowerby, (i.e., 38% sandstone, 62% limestone in Blind Beck compared to 28% and 69% respectively for Temple Sowerby). Although the time constant is similar for Temple Sowerby and Blind Beck, the pure time delay at Temple Sowerby is 5 h compared to only 1 h at Blind Beck. This may suggest that a similar pathway is dominant in both, but the response at Temple Sowerby has a delay in initial response due to channel routing from the rainy headwaters.

The observed and modelled discharge from the nested catchments Temple Sowerby, Great Musgrave and Kirkby Stephen is shown in Fig. 3 for the 4 month period from 1 December 2006 to 1 April 2007. Although a first order model cannot capture all the dynamics, in all these cases it models the rise and initial fall of the hydrographs well, explaining 84% of the observed data at Kirkby Stephen, 85% at Great Musgrave and 91% at Temple Sowerby. The long recessions are underestimated, (but are also underestimated even with a second-order model), indicating the presence of a very slow pathway, but one generating only a small proportion of the total discharge (i.e., by definition not a dominant runoff pathway). For this 4 month period the time constant increases as the catchment size increases, from 6.5 h at Kirkby Stephen to 16.3 h at Temple Sowerby, indicating that the dominant response at Temple Sowerby is slower than that at Kirkby Stephen.

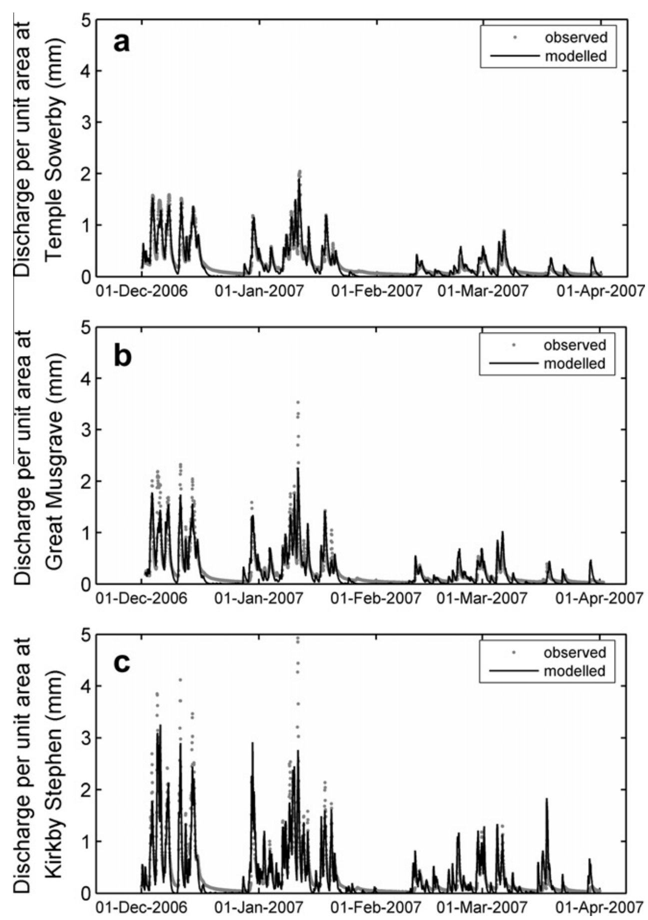

Fig. 3. Observed and modelled discharge using a first order linear model, December 2006–April 2007, (a) at Temple Sowerby, where a = [1.0–0.9405], b = [0 0 0 0 0 0.1161], R_T^2 = 0.91, YIC = −11.42, TC = 16.3 h; (b) at Great Musgrave, where a = [1.0–0.9226], b = [0 0 0 0.0757], R_T^2 = 0.85, YIC = −10.32, TC = 12.4 h; (c) at Kirkby Stephen, where a = [1.0–0.8574], b = [0 0 0.1161], R_T^2 = 0.84, YIC = −10.09, TC = 6.5 h. a and b are vectors of the coefficients in the polynomials in Eqs. (2) and (3); R_T^2 is the coefficient of determination; YIC is the Young Information Criterion; TC is the time constant.

The observed and modelled discharge at Temple Sowerby, Great Musgrave and Kirkby Stephen for the 8 day period 30 January–7 February 2004 is shown in Fig. 4. This multi-day flood event was responsible for extensive flooding of farmland in the Vale of Eden

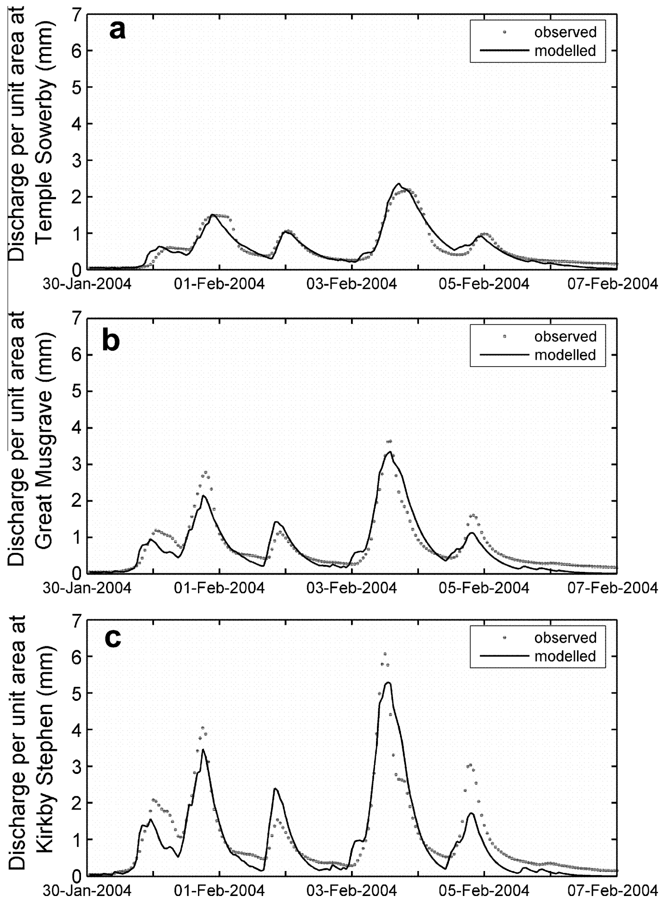

Fig. 4. Observed and modelled discharge using a first order linear model, 8 days from 30 January 2004, (a) at Temple Sowerby, where *a* = [1.0–0.9152], *b* = [0 0 0 0 0 0.0679], R_T^2 = 0.94, YIC = −10.09, TC = 11.3 h; (b) at Great Musgrave, where *a* = [1.0–0.8765], *b* = [0 0 0.1154], R_T^2 = 0.88, YIC = −8.67, TC = 7.6 h; (c) at Kirkby Stephen, where *a* = [1.0–0.8258], *b* = [0 0 0.2222], R_T^2 = 0.85, YIC = −8.05, TC = 5.2 h. *a* and *b* are vectors of the coefficients in the polynomials in Eqs. (2) and (3); R_T^2 is the coefficient of determination; YIC is the Young Information Criterion; TC is the time constant.

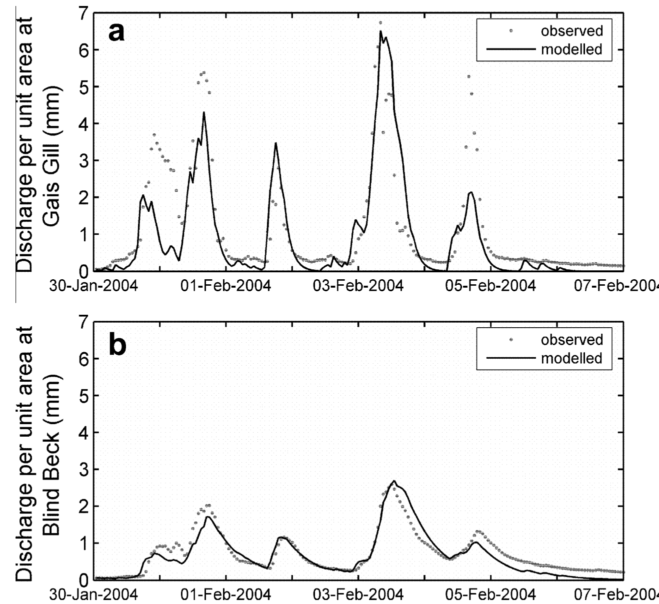

Fig. 5. Observed and modelled discharge using a first order linear model, 8 days from 30 January 2004, (a) at Gais Gill, where *a* = [1.0–0.6716], *b* = [0.4075], R_T^2 = 0.66, YIC = −5.70, TC = 2.5 h; (b) at Blind Beck, where *a* = [1.0–0.9126], *b* = [0 0.0781], R_T^2 = 0.86, YIC = −8.51, TC = 10.9 h. a and b are vectors of the coefficients in the polynomials in Eqs. (2) and (3); R_T^2 is the coefficient of determination; YIC is the Young Information Criterion; TC is the time constant.

and in the City of Carlisle (Mayes et al., 2006). To enable a direct comparison between sites for this event, the same rainfall input was used for all of them, the rainfall at Scale Beck. There may be a weak relationship between rainfall and altitude at the catchment scale, but this is unlikely to have an effect on the time constants, the purpose of this comparison. Comparison with the rainfall measured at a large number of rain gauges and averaged for each catchment using Thiessen polygons, as shown in Mayes et al. (2006), indicates that the rainfall pattern shown at Scale Beck is very similar to the catchment averaged pattern. Fig. 4 shows that a first-order, linear model identifies the dominant mode well for this storm event ($R_T^2 \geqslant 0.85$), in spite of the short time period. The damping of the hydrograph between Kirkby Stephen and Temple Sowerby is also visible.

Fig. 5 shows the observed and modelled discharge at Gais Gill and Blind Beck for the same 8 day period. The fit of the model at Gais Gill (R_T^2 = 0.66) is not as good as for the other sites, partly because of the smaller 'integrator effect' resulting from the smaller sub-surface storage.

For this event the same pattern of time constants is observed, with time constants increasing from 2.5 h at Gais Gill in the headwaters, to 7.6 h at Great Musgrave and 11.3 h at Temple Sowerby. Once again the time constant at Blind Beck (10.9 h) is most similar to Temple Sowerby.

Chappell et al. (2007) suggest that information about the dominant flow pathways can be drawn from knowledge of the presence or absence of permeable topsoil, permeable subsoil, permeable regolith and/or permeable rock. They suggest a conceptual model with four types of water pathway depending on the presence or absence of up to four types of strata (i.e., topsoil, subsoil, permeable regolith, permeable rock). A type I system has a thin layer of topsoil over impermeable rock. This would result in a very fast response in the hydrograph (time constant of minutes to a few hours). At the other end of the scale, a type IV system allows water movement through topsoil, subsoil and regolith to a deep permeable rock layer. This can result in a very slow response in the hydrograph (time constant of several months). The time constants for all of the catchments shown here suggest that most of the Upper Eden is Type II or III, with water flowing predominantly through the subsoil and/or regolith (except Gais Gill, which could represent Type I). In contrast, a Type IV system would be the River Lambourn in Berkshire UK, where the hydrograph shows a very slow response, with an annual cycle responding to winter rain. The River Lambourn is on a deep chalk aquifer and has long been a subject of study in the NERC LOCAR (Lowland Catchment Research) programme (e.g. Griffiths et al., 2006; Jackson et al., 2006; Wheater et al., 2007). For the Lambourn catchment gauged at Shaw (234.1 km^2, i.e. a similar size catchment to the Eden gauged at Great Musgrave) the same DBM modelling indicates a first order model with a pure time delay of 7 days and a time constant of 107 days. The model results illustrating this long time constant are shown in Fig. 6. Because of the long time constant of the dominant mode it was necessary to model a period of data covering several years. The rainfall used in this case was the residual rainfall after an evaporation component had been removed. A daily time step was sufficient and very similar results were obtained with a 10 day time step. The model fit is less good in the drier years (e.g. 2004 and 2005); however, the results are only included here as a reference for a catchment with a time constant a factor of 100–500 times larger than any of those in the Upper Eden.

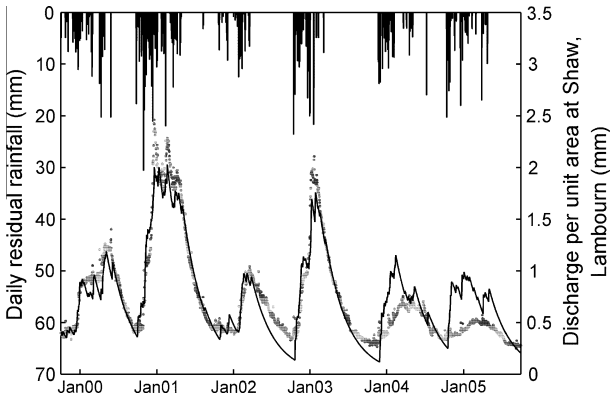

Fig. 6. Observed (grey dots) and modelled (black line) discharge at Shaw, River Lambourn, Berkshire (234.1 km^2), using a first order model with a one day time step, October 1999–September 2005. Time constant = 107.1 days; pure time delay = 7 days.

3.2. Time constants and pure time delays within Blind Beck

Rainfall–runoff in the catchment of Blind Beck was also compared with that in its 1.0 km^2 Low Hall sub-catchment. The low flow in Low Hall stream had a much higher specific conductivity than in Blind Beck, which was due to a higher concentration of calcium carbonate in the water, suggesting that the water in Low Hall stream spent longer in the ground, with longer contact with the rock. In addition, Low Hall stream was warmer than Blind Beck in winter and colder than Blind Beck in summer (Ockenden, 2010). This is consistent with Low Hall stream having a greater input from a deeper source with slower velocities and greater storage. DBM modelling was used to investigate whether this difference was identifiable from the rainfall–runoff response.

In contrast to the other catchments in the Upper Eden, an acceptable first order model ($R_T^2 > 0.6$) could not be identified for the Low Hall catchment. In this case a second-order model was identified, with a [2 2 δ] structure, where δ is the pure time delay. Using the method of decomposition into partial fractions (Beven, 2001), this was interpreted as two parallel pathways.

For comparison with Blind Beck, the same time period was modelled for both Low Hall and Blind Beck, using a [2 2 δ] model for both, as this was the simplest acceptable model identified for Low Hall. For both Blind Beck and Low Hall the time delay was 1 h (i.e., 1 time step). The observed and modelled discharges are shown for both Blind Beck and Low Hall in Fig. 7. Compared to a first order model of Blind Beck, the second-order model captures the latter part of the recessions much better, which results in a slightly better R_T^2 value (0.91 compared to 0.88 for the same period at Blind Beck), but a less negative YIC (−9.5 for the second-order model compared to −10.3 for the first order model) due to the increase in number of parameters to be identified.

Decomposition of each model into two parallel paths gives the results shown in Fig. 8. The time constant for the slow pathway at Low Hall stream is 251.6 h, with approximately 76% of the water taking this path. In comparison, the time constant for the slow pathway at Blind Beck is 60.3 h with only 46% of water taking this path. Thus the model supports the evidence that more of the water in Low Hall stream has taken a slower and possibly deeper pathway.

In contrast, if a [2 2 δ] model is used to model the runoff at Temple Sowerby for the same period, the model decomposition gives a fast time constant of 10.6 h, with 70% taking this pathway and a slow time constant of 138.9 h with 30% taking this pathway. Compared to Blind Beck, although the fast time constant is longer, this is offset by more water taking this route, so differences are not clear.

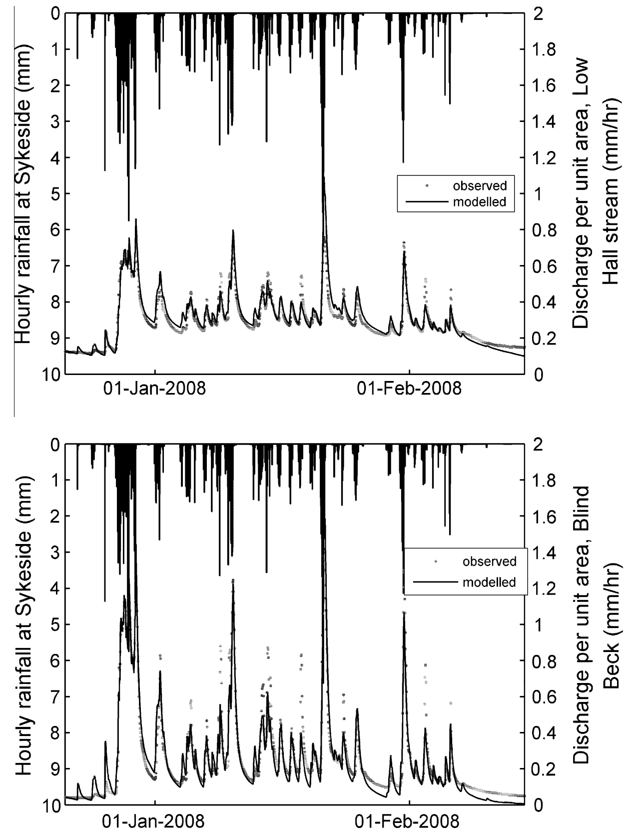

Fig. 7. Observed and modelled discharge in Low Hall stream and Blind Beck, for the period 21 December 2007–15 February 2008, using a second order [2 2] model.

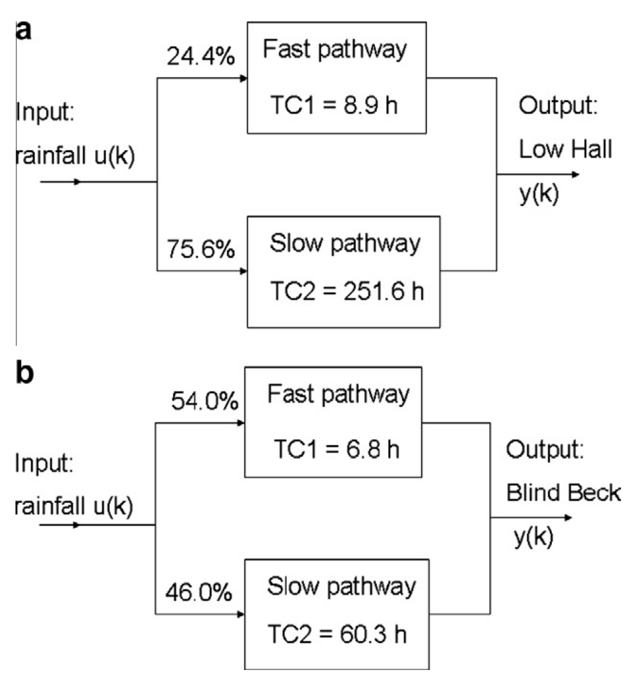

Fig. 8. Decomposition of second-order model into two parallel pathways for (a) Low Hall stream (1.0 km^2) and (b) Blind Beck (8.8 km^2).

The uncertainty in the time constants and percentages on each path for these second-order models was investigated with a Monte Carlo analysis (with 10,000 realisations) using the uncertainty on the model parameters, estimated at the time of model identification. The 5% and 95% confidence limits on the time constants and

Table 4
5% and 95% confidence limits for time constants and percentages on each pathway for Low Hall, Blind Beck and Temple Sowerby, from Monte Carlo analysis.

	Fast pathway		Slow pathway	
	TC1 (h)	%	TC2 (h)	%
Low Hall Stream	8.5–9.3	23.8–25.0%	243.0–259.7	74.9–76.2%
Blind Beck	6.6–7.1	44–66%	56.5–64.7	34–56%
Temple Sowerby	10.3–10.9	63–78%	132.9–145.5	22–37%

pathway percentages for Low Hall, Blind Beck and Temple Sowerby for the period modelled above are shown in Table 4.

The presence of a slower pathway in the runoff dynamics of Low Hall stream is confirmed by this Monte Carlo analysis. The smaller range of uncertainty in the percentages on each path for the Low Hall model is due to better model identification. For both Blind Beck and Temple Sowerby, the data suggest a first order model for least parameter uncertainty, but a second-order model is necessary for comparison with Low Hall. Although the second-order models also fit the data well, the parameter uncertainty is increased.

4. Conclusions

Data-based mechanistic modelling was shown to be a useful tool for investigating spatial variations in runoff dynamics in the Upper Eden catchment, UK. A first order linear model adequately identified the dominant mode in the sub-catchments of Gais Gill, Kirkby Stephen, Great Musgrave, Temple Sowerby and Blind Beck. A consistent pattern of dynamic response characteristics (notably the time constant) between catchments was observed. In the nested catchments, time constants increased as the catchment size increased, because the sub-surface and channel routing distances increased. However, the 8.8 km^2 Blind Beck catchment had time constants most similar to the 616 km^2 Temple Sowerby basin. This was attributed to the relatively high proportion of permeable sandstone underlying both catchments. Thus the proportion of a catchment on permeable rock aquifer can become as important a factor as the catchment size (that affects the sub-surface and channel routing times).

The pure time delay was 5 h at Temple Sowerby compared to 1 h at Blind Beck. This suggested that the response at Temple Sowerby could be due to the effect of channel routing in response to rainfall primarily in the headwater sub-catchments.

A first order model could not be identified for the 1.0 km^2 sub-catchment of Low Hall stream that was entirely on permeable sandstone overlain by Quaternary deposits, including a high proportion of coarse river terrace deposits. A second-order model identified a higher proportion of water taking a slow pathway in Low Hall stream (probably via the greater storage afforded by the greater proportion of the deeper rock pathway) compared with the water in Blind Beck. The presence of the more permeable drift cover in the Low Hall sub-catchment compared to that in Blind Beck may be an important factor in allowing the deep groundwater to return to the surface in this sub-catchment.

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