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What really happens at the end of the rainbow? – paying the price for reducing uncertainty (using reverse hydrology models)

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Abstract

Modelling of environmental processes is subject to a high degree of uncertainty due to the incorporation of random errors and a lack of knowledge about how processes operate at the scale of interest. Use of uncertain data when identifying and calibrating a model can lead to disinformative data being included in the procedure, resulting in uncertain parameter estimation and ambiguity in the outcomes. Rainfall-runoff modelling where a single rain-gauge is often assumed to be representative of the potentially highly variable (in both space and time) rainfall field is a good example. The noisy pattern of rainfall inputs is transformed by the catchment into streamflow. The streamflow pattern is dependent on the spatio-temporal pattern of rainfall and of the dominant processes operating within the catchment. Inverse modelling of the catchment dynamics, that is, inferring catchment rainfall from streamflow, provides a possible means of improving the estimated rainfall input because all rain falling on the catchment becomes streamflow, and thus, providing improved forecasts of the streamflow output. A combination of inverse modelling, time series analysis, spatial analysis and spectral analysis may also help to provide an insight into the complex processes operating within the catchment system. This paper applies a novel method for inferring true catchment rainfall from streamflow highlighting that the streamflow is better estimated using inferred rainfall than observed rainfall (from a single gauge) because a single gauge only gives a partial description of the rainfall field. However reducing uncertainty in this way comes at a price, in this case, the reduction in time-resolution of the inferred rainfall series.

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1. Introduction

Rainfall is the most important input to most hydrological models. The rainfall field is variable in both time and space and thus has inherent uncertainty. This is amplified by the fact that rainfall measured at a point by a single rain gauge is often assumed to be representative of a catchment many kilometres squared in area. Figure 1 shows the variability in the rainfall field across the Brue catchment (in the south-west of the UK) measured at 15-minute intervals by a 23-gauge network. The pie charts at the top right of each image indicate how many gauges are measuring rainfall at each time interval – the more yellow, the more gauges have rainfall. Estimates used to design flood defences are based on historic records of rainfall and streamflow which are subject to uncertainty from many sources including measurement techniques, instrumentation, changes to the system, model structure, lack of understanding of the processes at the scale of interest and those all-important unknowns some of which are ‘unknown unknowns’. The ideal would be to provide a 100% certain forecast of the future. However uncertainty dictates that this is unlikely to happen. The best that can be done is to strive for improved understanding of the processes and better measurement techniques that will help to reduce uncertainty.

So how can rainfall estimates be improved? As can be seen, rainfall is variable in time and space (figure 1) and a single gauge may not be representative of the rain that falls over the whole catchment, however, all the rain that falls on the catchment is integrated, by the active processes, into streamflow that can be measured at the catchment outlet. Working backwards from the measured streamflow, it should be possible to use the information in the streamflow to estimate the rainfall.

Considerable interest has been shown in ‘reverse hydrology’ in recent years. Although streamflow is itself subject to uncertainty, it is assumed that errors in measurement are much smaller than the errors in estimates of

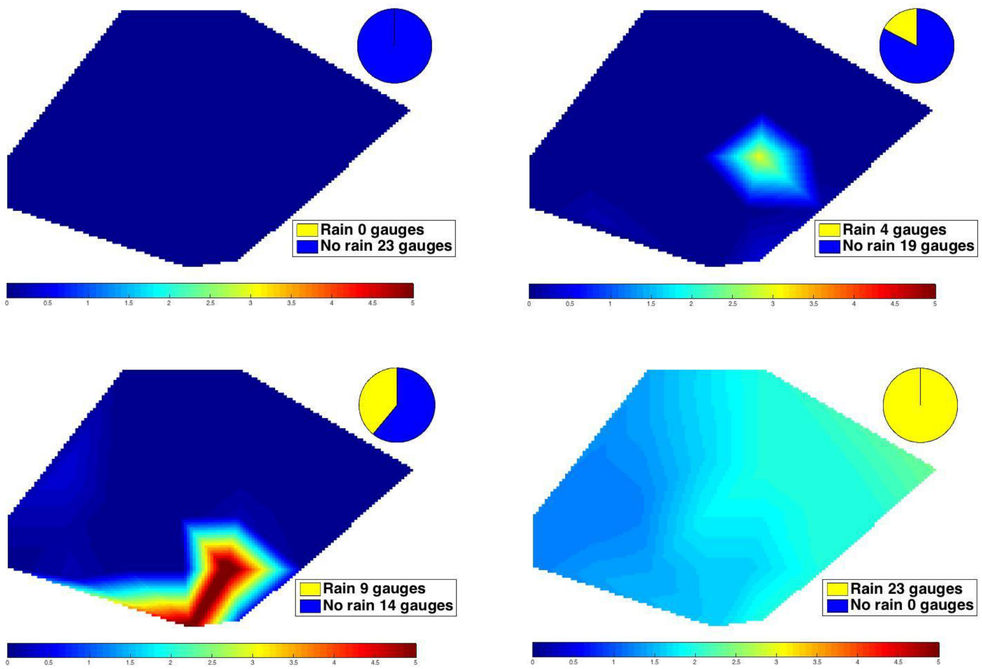

Figure 1 – The variability in the rainfall field in space and time over the Brue catchment – brighter colours mean more rain.

catchment rainfall [1] especially in mountainous catchments where altitude also plays a role in rainfall distribution. Studies in this area include those by Croke [2, 3], Kirchner [4], Andrews *et al*. [5], Young and Sumisławska [6], Brocca [7, 8] and Kretzschmar *et al*. [9, 10]. Reverse hydrology could be an important tool in promoting understanding of catchment rainfall distribution and the processes by which it is converted to streamflow and the identification of periods of inconsistent input-output data.

2. Methodology

Distributed models that attempt to take account of the variations in rainfall and catchment characteristics have a large number of parameters that must be estimated in order to fit the model. Many of these parameters have no physical meaning or lose their meaning in the calibration process when adjusted to make the model outputs a better fit to the observed measurements. Given the uncertainty involved at all stages of the process, it is hard to justify these highly parameterised models though they have a place in attempting to explain the processes involved. This study uses the Data Based Mechanistic (DBM) modelling approach [11], which allows the data to suggest the form of the model. Several models that fit the data well may be identified (the equifinality concept described in [12]) but only those that have few parameters (are parsimonious) and have a physical interpretation are accepted.

The method presented here uses systems analysis techniques, implemented using the Captain toolbox in Matlab [13] to identify a continuous time (CT) transfer function model utilising the high-resolution data (in this case, rainfall and streamflow) needed to capture the dynamics of the catchment. The model (or models) thus identified can be inverted using a using a novel regularisation process detailed in [9]. The output from the regularisation process is an inferred rainfall series. The transfer function model is a linear relationship so non-linearity is modelled as a separate process as shown in figure 2. Advantages of using a CT formulation are that a wide range of catchment dynamics can be modelled and the parameters have a direct physical interpretation that is independent of the sampling rate [14]. The inversion itself is necessarily badly posed due to the need to invert processes which have been integrated in both time and space. However, applying the regularisation technique to CT models makes inversion possible without the amplification of the noise present in the data as is the case with direct inversion of the transfer function [5, 9]. Kirchner's method [4] links rainfall and streamflow through a storage sensitivity function but it is limited to simple catchments that behave as single-reservoir (first-order) systems [1]. Discussion of other approaches is made in the referenced literature [2, 3, 4, 5, 6, 7, 8, 9, 10]. Working with sub-hourly data from two contrasting catchments, Kretzschmar *et al.* [9] showed that while the direct inverse of a transfer function produced an inferred effective rainfall series characterized by high frequency noise components, the regularisation process produced a much smoother rainfall profile sacrificing time resolution in favour of numerical stability. They also showed that both rainfall sequences resulted in similar modelled flow sequences, which fitted the observed streamflow data more closely than flow modelled using the observed rainfall implying that the dynamics of the catchment were being effectively captured in both cases. The high frequency behaviour of the direct inverse method has no physical interpretation so can be deemed to fail the criteria of the DBM methodology. Further investigation [10] made use of sub-sampling and spectral analysis to quantify the loss of resolution and showed that the inferred rainfall sequences were still able to capture the catchment dynamics.

Catchments integrate the rainfall falling on them in space as well as time when converting the rainfall into flow. This paper presents an initial investigation of spatial uncertainty utilising the inverse regularisation method outlined above.

3. Test catchment

This paper utilises the heavily instrumented Brue catchment in South-west England. It has 49 rain gauges in an area of 135.2 km^2 enabling spatial variability to be investigated. There is an elevation change of approximately 300m from south-west to north-east across the catchment. The underlying geology is a combination of mudstone and limestone with a limestone ridge running in an arc across from north to south across the eastern upland area (see figure 3). The catchment can largely be split into impermeable low-land to the west, higher-land to the east where

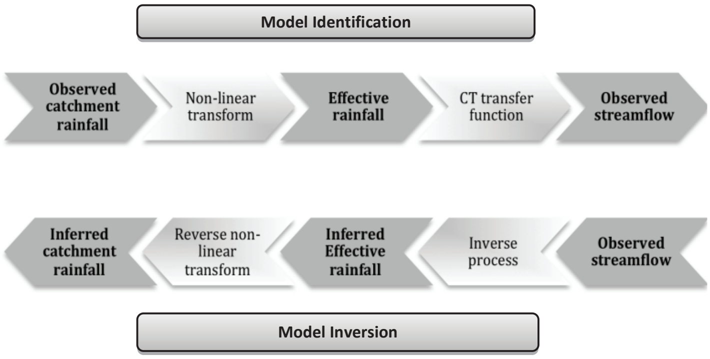

Figure 2 - the model identification and inversion workflow showing the off-line non-linear transformation

the limestone ridge is permeable, and the far east of the catchment which is largely impermeable. Land use is mostly pasture on clay soils with some woodland on the elevated eastern side [15]. The Brue research catchment was set up in 1993 as part of a Natural Environment Research Council (NERC) special topic research programme – the Hydrological Radar Experiment (HYREX) [15]. It ran for three years but the data has been extensively used in many subsequent research projects [e.g. 15, 16, 17, 18, 19].

4. Initial spatial analysis

Due to the geographical proximity of many of the gauges in the Brue catchment, the most highly correlated gauges were rejected and a network of 23, retaining the geographical spread, was chosen for analysis – the reduced network is shown in figure 3 against the underlying geology of the catchment. A detailed analysis of the effect of the number of gauges and the ability of the inversion process to highlight disinformative sections of data is planned. Early results comparing the results from the full gauge set (49F) with the reduced gauge set (23R) and individual gauges is presented here. Two basic methods of averaging are investigated – simple arithmetic averaging (AV) and Thiessen polygons (TP) where the gauges are weighted by their area of influence. Both methods are well documented (e.g. [20, p166]). Given the number of gauges, the effect of elevation is not included per se as it is expected to vary from event to event.

Firstly the set 23R was compared to the 49F using the GORE (Goodness of Rainfall Estimate - i.e. how well the sub-sample represents the true rainfall) and BALANCE (a measure of over/under estimation of the sub-sample with respect to the true rainfall) metrics presented by Andréassian *et al.*[21] and shown in equations (1) and (2).

$$GORE = 1 - \frac{\sum\left(\sqrt{ER} - \sqrt{TR}\right)^2}{\sum\left(\sqrt{TR} - \overline{\sqrt{TR}}\right)^2} \qquad (1)$$

where ER is the sample rainfall in a single time-step and TR is the corresponding observation from the true (or reference) rainfall. GORE can vary between $-\infty$ and 1 where 1 indicates that the sub-sample (ER) perfectly represents the true rainfall (TR).

$$BALANCE = \frac{\sum ER}{\sum TR} \qquad (2)$$

If BALANCE > 1, the sub-set over-estimates, if BALANCE < 1, the sub-set under-estimates and a value close to 1 indicates a good fit.

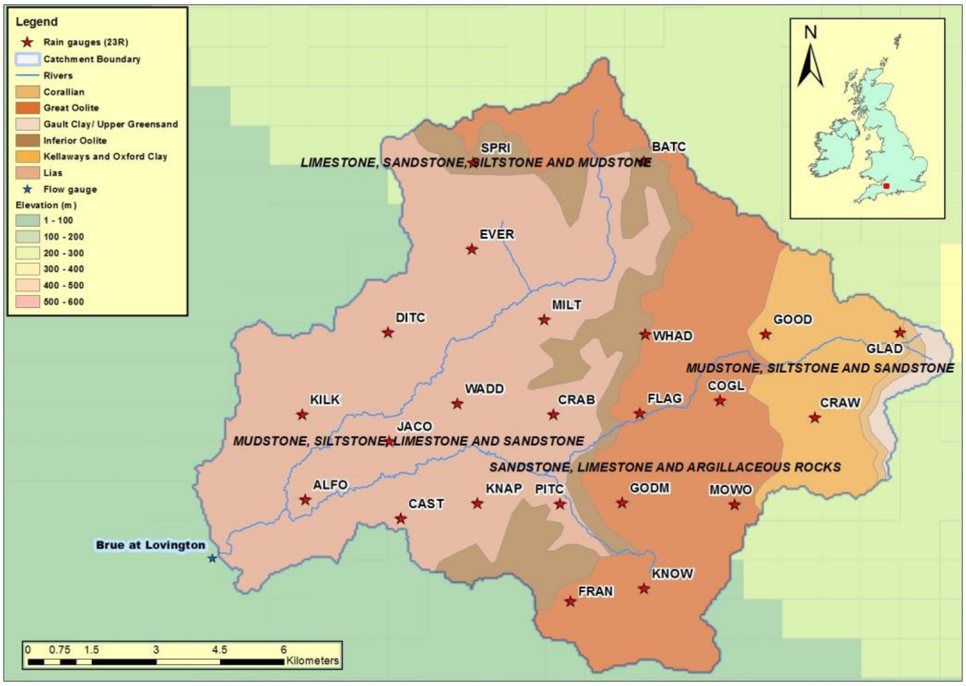

Figure 3 - Brue catchment geology, location and gauge network. (Crown Copyright/database right 2016. A British Geological Survey/EDINA supplied service; National River Flow Archive [22])

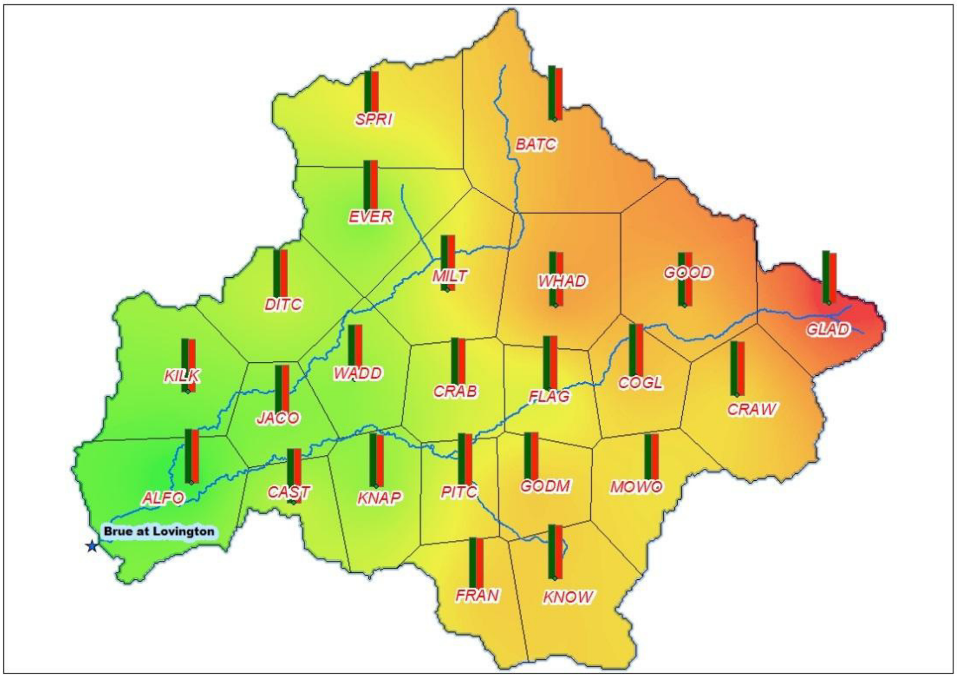

Figure 4 - Comparison of modelled R_t^2 (green bars) with inferred, aggregated R_t^2 (red bars) for each individual gauge. (Crown Copyright/database right 2016. A British Geological Survey/EDINA supplied service; National River Flow Archive [22])

Table 1 shows the comparison between the TR (49F) and ER (23R) for the two averaging methods and indicates that the reduced network (23R) is a good estimator of the TR as estimated using the full gauge set (49F). For this catchment and gauge set, there is little to choose between the methods. Gauges drawn from set 23R (Thiessen polygon method) are thus used to estimate the average CT transfer function for the catchment.

Table 1 - Validation of the 23 gauge network with respect to the full 49 gauges using the BALANCE and GORE criteria.

	BALANCE	Percentage over/under estimation	GORE
Arithmetic average	1.005	+0.5%	0.985
Thiessen Polygon	1.004	+0.4%	0.987

5. Initial Results and Discussion

Models were identified using the observed rainfall series for each individual gauge drawn from set 23R and the catchment outflow then inverted using the regularisation method. In order to compare the inferred and observed rainfall sequences and determine the time resolution of the inferred sequence, aggregation by sub-sampling at increasing sampling intervals was performed. Nash-Sutcliffe Efficiency (R_t^2) was calculated at each interval and the time interval with the closest fit to the observed (aggregated) rainfall (highest R_t^2) was taken to be the time resolution of the inferred rainfall [10]. The R_t^2 of the aggregated sequence was compared with the R_t^2 of the fitted model and the results plotted in figure 4 indicating that, despite the loss of time-resolution, the results are closely comparable. For all gauges, the aggregation period (estimate of time resolution) of the inferred rainfall sequence is less than fast time constant (TCq) implying that the catchment dynamics are being captured. Flow was generated using the inferred rainfall sequence from each individual gauge. The resulting flow sequences were found to more closely match the observed flow (typically $R_t^2 = 0.996$) than flows generated from models fitted to individual gauges ($R_t^2 = 0.804$ to 0.831) or flow generated from a model fitted using the catchment average rainfall calculated from the 23R (TP average) gauge set ($R_t^2 = 0.852$). This is consistent with the results presented in [10].

Examination of the observed rainfall (top plots in figure 5) shows the variability of the rainfall field across the catchment and emphasises that it can be raining hard in one place whilst it is dry in another (see also figure 1) resulting in artificial spikes in the generated flow - particularly evident in the plot for KILK, one reason why some events can be disinformative when used for model calibration [23]. Further work is planned to investigate the effects of different densities and numbers of rain gauges, identification of disinformative periods of data, and how it might be possible to measure how representative individual gauges or gauge sets are of the catchment as a whole.

6. Conclusions

As has been demonstrated, reverse hydrology utilises the information in the streamflow exiting the catchment to infer the rain that has fallen over the whole catchment rather than the amount measured at an individual rain gauge

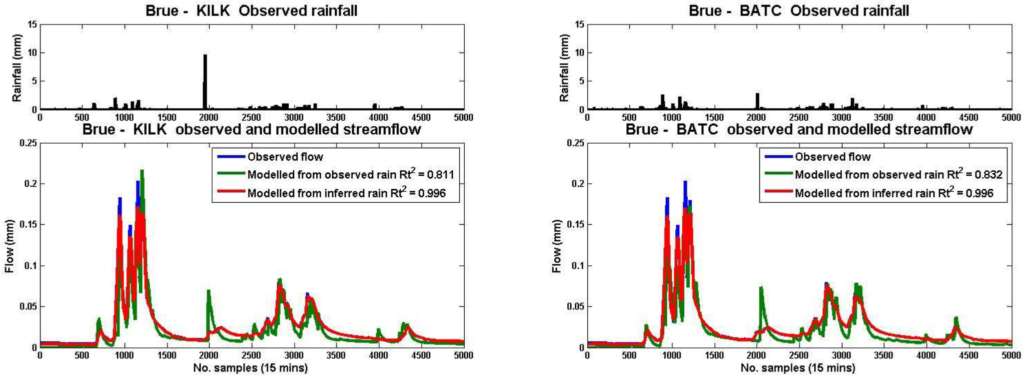

Figure 5 - Observed rainfall from example gauges comparing flow generated using the observed and inferred rainfall with the observed flow.

where the latter may not be representative of the total rainfall field and may even lead to spurious spikes in the modelled flow where rain has been measured at the gauge but not elsewhere in the catchment. This technique could deliver an improved estimate of the total rainfall. However, the reduction in uncertainty in the rainfall estimates comes at a price – a reduction in the time resolution of the rainfall series. As has been demonstrated, this is not a problem as long as the resolution is still fine enough to capture the dynamics of the catchment.

Reverse hydrology could be an important tool in developing understanding of catchment rainfall distribution and the processes by which it is converted into streamflow leading to a reduction in uncertainty and an improvement of future flow predictions that might result in saved lives, reduced damage to property and infrastructure and ultimately to decreased costs.

Acknowledgements

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