Environmental Modelling & Software 60 (2014) 290–301

Contents lists available at ScienceDirect

Environmental Modelling & Software

journal homepage: www.elsevier.com/locate/envsoft

Reversing hydrology: Estimation of sub-hourly rainfall time-series from streamflow

Ann Kretzschmar, Wlodek Tych*, Nick A. Chappell

Lancaster Environment Centre, Lancaster University, Lancaster LA14YQ, UK

A R T I C L E I N F O

Article history:
Received 18 December 2013
Received in revised form
13 June 2014
Accepted 21 June 2014
Available online

Keywords:
Hydrology
Inverse modelling
Transfer function
Regularisation
Continuous time
Data-based mechanistic modelling
Hyetograph
Hydrograph
Derivative estimation
Input estimation

A B S T R A C T

A novel solution to the estimation of catchment rainfall at a sub-hourly resolution from measured streamflow is introduced and evaluated for two basins with markedly different flow pathways and rainfall regimes. It combines a continuous-time transfer function model with regularised derivative estimates obtained using a recursive method with capacity for handling missing data. The method has general implications for off-line estimation of unknown inputs as well as robust estimation of derivatives. It is compared with an existing approach using a range of model metrics, including residuals analysis and visuals; and is shown to recover the salient features of the observed, sub-hourly rainfall, sufficient to produce a precise estimate of streamflow, indistinguishable from the output of the catchment model in response to the observed rainfall data. Results indicate potential for use of this method in environment-related applications for periods lacking sub-hourly rainfall observations.

1. Introduction

Accurate simulation of stream hydrographs is strongly dependent on the availability of rainfall data at a sufficiently high, sub-daily sampling intensity (Hjelmfelt, 1981; Littlewood and Croke, 2013). Additionally, hydrograph simulation may be sensitive to the spatial intensity of rainfall sampling (Ogden and Julien, 1994; Bardossy and Das, 2008) or to the uncertainties arising from local calibrations of rainfall radar (Cunha et al., 2012) or individual raingauges (Yu et al., 1997). Despite this importance, most gauged basins lack the necessary long-term, sub-hourly rainfall records (and adequate spatial rainfall sampling) to combine with the streamflow records that are, by contrast, typically monitored at sub-hourly intervals for several decades. If those short-term rainfall characteristics responsible for producing stream hydrographs (see Eagleson, 1967; Obled et al., 1994) can be estimated from streamflow, the resultant synthetic rainfall series may be useful in many applications. For example, synthetic rainfall records could be derived for basins with long-term streamflow, but only short-term rainfall, to: (1) evaluate long-term, rainfall estimates from Global Circulation Models for specific catchments (see Fujihara et al., 2008), (2) provide long-term rainfall records for long-term aquatic ecology studies (e.g., Ormerod and Durance, 2009), and (3) identify localised rainfall cells or snowfall events that affect the streamflow but are poorly represented in raingauge records (Kirchner, 2009).

This study uses a Data-Based Mechanistic (DBM) modelling approach to identify linear Continuous-Time Transfer Function (CT-TF) models (Young and Garnier, 2006) between sub-hourly rainfall and streamflow. These forward CT-TF models are then inverted to derive rainfall time-series using a novel method that utilises regularisation techniques. Algorithms within the CAPTAIN Toolbox (Taylor et al., 2007) are used for this modelling and the methodology evaluated by application to two micro- or headwater-catchments with contrasting rainfall and response characteristics, namely the humid tropical Baru catchment and the humid temperate Blind Beck catchment. Classical rainfall–runoff non-linearity utilises a power law relationship between measured and effective rainfall (Beven, 2011) implemented as a Hammerstein type non-linearity (Wang and Henriksen, 1994) separated from the linear dynamics of the transfer function. As the power function is monotonic, it is easily inverted, making it trivial to apply in

* Corresponding author. Tel.: +44 1524 593973.
E-mail address: w.tych@lancaster.ac.uk (W. Tych).

http://dx.doi.org/10.1016/j.envsoft.2014.06.017

combination with the effective rainfall estimate generated by the proposed method as illustrated in Fig. 1.

The graphical expression of the forward CT-TF model of a rainfall–streamflow response in discrete time is the impulse response function and this is directly equivalent to the unit hydrograph or UH developed by Sherman (1932). Inversion of the UH or its CT-TF equivalent to derive rainfall from streamflow has been attempted by Hino (1986), Croke (2006), Kirchner (2009), Andrews et al. (2010) and Young and Sumisławska (2012). These studies have used a range of different approaches. For example, Hino (1986) applied a standard regularised Least Squares (LS) solution to the inversion of a catchment model of ARX form (i.e., autoregressive with exogenous variables: see Box et al., 2008). This approach differs from the CT-TF based approach proposed here, in that potentially huge matrix inversions are needed. Kirchner (2009) used a very different method that involved the construction of a first-order, non-linear differential equation linking rainfall, evaporation and streamflow through the sensitivity function, resulting in a compound measure of precipitation and evaporation, which is then reduced to rainfall through making assumptions about the relationship between the rainfall and residual rainfall (i.e., rainfall minus evaporation). Kirchner's method has been applied to the Rietholzbach catchment in Switzerland (Teuling et al., 2010) and to 24 diverse catchments in Luxembourg (Krier et al., 2012) where it reproduces the streamflow and storage dynamics for catchments characterised by a single storage–discharge relationship but cannot explain more complex travel times. Andrews et al. (2010) used inverse filtering, applying similar CAPTAIN modelling methods to the ones proposed here, but using a direct inverse transfer function in discrete time. As this is methodologically the nearest approach to the proposed one and, at the same time, highlights the practical problems with direct inversion of transfer function models, it was chosen as a comparison in this study. Young and Sumisławska (2012) applied non-minimal state-space feedback control methods to inversion of discrete time transfer function models, based on the work of Antsaklis (1978).

Jakeman and Young (1984) were the first to indicate that recursive regularisation might be a useful approach to derive rainfall time-series from the UH, but without offering an implementation of the algorithm or examples. The novel method proposed here has been developed by combining these ideas with developments in the identification of CT-TF models (e.g., Young and Garnier, 2006) and improvements in the CAPTAIN routines (Taylor et al., 2007). The inverse process is based on differentiation (Young, 2006), and so may be expected to be ill-posed and sensitive to noise in the streamflow data (O'Sullivan, 1986; Neumaier, 1998; Tarantola, 2005). The direct inverse of the discrete transfer function method involves differencing, the key issue addressed in the proposed method by using regularised derivatives, potentially its major advantage.

The generality of our approach indicates that it could be used within any modelling framework involving DBM or top-down catchment modelling. Integrating it within other frameworks, for instance to assess the information content of hydrological data (Beven and Smith, 2014) is already a part of an existing project which partly funded this study (NERC CREDIBLE project – see Acknowledgements for details). Another good example of the use for this approach would be within the **hydromad** framework (Andrews et al., 2011) where it could be a part of either model or data evaluation process. Such application could be based on the reasoning that a model and data combo (the principle of DBM approach), which invert well should be more reliable (this assertion will be the subject of future work). Within the same **hydromad** framework a similar reasoning could be used to verify the placement of raingauges within a catchment. If the inversion generates poorly fitting inferred rainfall with many negative periods it could indicate that the present raingauges do not provide full information about the catchment rainfall due to their placement. Andrews et al. (2011) also indicate the use of such inversion routines in calibration of full hydrological models.

Reaching further out, beyond the discipline of hydrology, there are many other situations where either input estimation of a dynamic system (e.g., Maquin, 1994; Yang and Wilde, 1988 and many others), or more generally, robust derivative estimation problems (De Brabanter et al., 2011) could benefit from the solution provided here. The off-line character of the method, characteristic for regularisation-based methods, excludes on-line applications, such as input observers in control engineering, but provides more flexibility, for instance by easy compensation of pure time delays in the transfer functions.

2. Novel parsimonious method for input estimation using reduced order output derivatives

To obtain a well-defined and effective inverse of any transformation (e.g., UH or equivalently a TF), the transformation itself must be well defined. It must capture the character of the system without any unnecessary complexity that would result in the transformation itself being ill-defined. This is the essence of the philosophy of the Data-Based Mechanistic (DBM) approach of Young (1998, 1999) that aims to produce models that fit the data well with as few parameters as are necessary to capture the dominant dynamic modes of the system. CAPTAIN tools are used to identify models using this underlying philosophy.

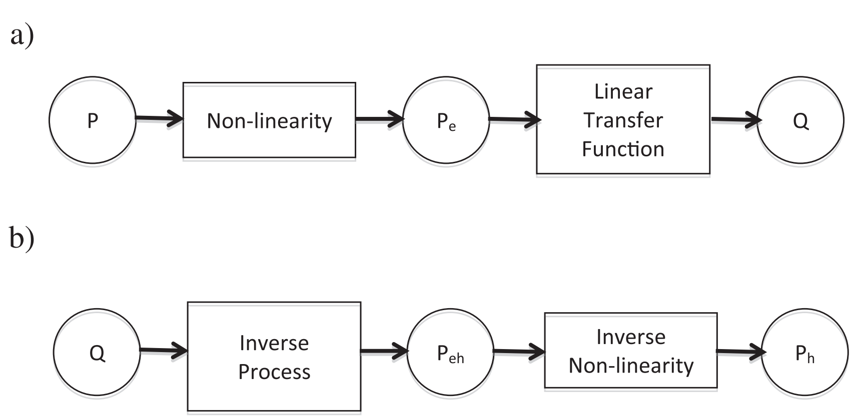

Fig. 1. The use of Hammerstein-type non-linearity in the model identification (a) and inversion (b) processes where P is the observed rainfall, P_e is the effective rainfall, Q is the observed streamflow, P_{eh} is the inferred effective rainfall and P_h is the inferred rainfall with the non-linearity reapplied.

The relationship between rainfall and streamflow expressed as a purely linear CT-TF may be given by:

$$Q = \frac{\beta_0 s^m + \beta_1 s^{m-1} + \cdots + \beta_m}{s^n + \alpha_1 s^{n-1} + \cdots + \alpha_n} e^{-s\tau} R \quad (1)$$

where Q and R are Laplace transforms of $Q(t)$ (streamflow) and $R(t)$ (discharge), s^r is the Laplace operator for r^{th} time derivative, ($s^r = \mathrm{d}^r/\mathrm{d}t^r$), $e^{-s\tau}$ is the Laplace transform for pure time delay between rainfall and the initial streamflow response τ, with the model parameter vector: $\theta = [\alpha_1\,\alpha_2 \cdots \alpha_n\,\beta_0\,\beta_1 \cdots \beta_m]^T$ of dimension $n+m+1$. These parameters are estimated from the data along with their covariance matrix, $\mathbf{C}_\theta$, using the Refined Instrumental Variable (RIV) method (Young and Jakeman, 1980) within the CAPTAIN toolbox. With CT-TFs, fast responding modes of catchment response can be estimated at the same time as very slow modes; one of their key advantages over discrete time approaches. Systems with widely-spaced time constants ('stiff systems') are known to be difficult to handle numerically including estimation of their parameters.

By its very nature (i.e., point measurements of rainfall), a transfer function model encapsulates both temporal and spatial modes of integration of the rainfall by the catchment. The inverse relationship expressing the streamflow-derived rainfall using the transfer function equation (2) will have the general form of:

$$\hat{R} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}{s^m + a_1 s^{m-1} + \cdots + a_m} e^{s\tau} Q \quad (2)$$

where $ai = \beta_i/\beta_0,\ i = 1, \ldots, m$ and $bi = \alpha_i/\beta_0,\ i = 1, \ldots, n$ to ensure the denominator polynomial is monic, with $n \geq m$ as in Eq. (2). The negative time delay is accounted for by off-line data-offset adjustment. The ill-posed nature of this inverse relationship is aggravated by the fact that often n is greater than m by more than one, reflecting the strong integrative character of catchment systems. This results in pure derivatives of the output that are often of an order higher than one (see Eq. (2)). Indeed most software environments such as Matlab do not even allow simulation of such systems, labelling them as improper. It should be noted here that the danger of obtaining unstable inverse models when the original model is non-minimum-phase (i.e., has zeroes in the right half-plane) is avoided altogether, as the DBM modelling methodology means that such models will be rejected at an early stage as non-physical.

The proposed solution, illustrated in Equation (3), consists of using regularised derivative estimates that is consistent with, but extending the approach proposed by Jakeman and Young (1984), namely:

$$\hat{R}\, e^{-s\tau} = \frac{b_0 \left\{ \widehat{s^n Q} \right\}^* + b_1 \left\{ \widehat{s^{n-1} Q} \right\}^* + \cdots + b_n Q}{s^m + a_1 s^{m-1} + \cdots + a_m} \quad (3)$$

where $\{\widehat{s^n Q}\}^* = \mathcal{L}\{\mathrm{d}^n \widehat{/\mathrm{d}t^n} Q\}$ is the Laplace transform of the optimised regularised estimate of the n^{th} time derivative of Q: $\mathrm{d}^n/\mathrm{d}t^n Q$.

Note that for $n > m$ this equation is equivalent to

$$\hat{R}\, e^{-s\tau} = \frac{b_0}{A(s)} \left\{ \widehat{s^n Q} \right\}^* + \frac{b_1}{A(s)} \left\{ \widehat{s^{n-1} Q} \right\}^* + \cdots + \frac{b_{m+1}}{A(s)} \left\{ \widehat{s^{m+1} Q} \right\}^* + \frac{b_m s^m + \cdots + b_n}{A(s)} Q \quad (4)$$

where:

$$A(s) = s^m + a_1 s^{m-1} + \cdots + a_m \quad (5)$$

In the latter, the final component is a proper transfer function, the preceding components are weighted (by $b_0 \cdots b_{m-n}$ respectively) regularised derivatives of order $n \cdots m+1$, all of them filtered with $A(s)$. It is worth noting that because of the filtering, the nth regularised derivative estimate is not indeed required, instead the $((n-m)^{\text{th}}, \ldots, 1^{\text{st}})$ order regularised derivative filtered with proper transfer functions is used, as shown below:

$$\frac{b_0}{A(s)} \left\{ \widehat{s^n Q} \right\}^* \approx \frac{b_0 s^m}{A(s)} \left\{ \widehat{s^{n-m} Q} \right\}^* \quad (6)$$

Equation (4) (with substitution based on Equation (6)) can be interpreted as a bank of filtered regularised derivatives added together, weighted by the inverse TF numerator coefficients b_0, b_1, …, b_n. In practical implementation therefore, the number of regularised derivatives estimated is limited to the difference between the orders of the numerator and the denominator of the original transfer function (3), *i.e.*, $(n-m)$, as the remaining derivatives are used implicitly in their filtered form making the algorithm more robust than its alternatives using a discrete transfer function inverse. Use of regularisation results in a trade-off between moderating the noise-amplifying ill-effects of the inversion process, and of the temporal resolution of the resulting rainfall time-series estimated. In order to obtain regularised estimates of derivatives of streamflow time-series up to order $n-m$, the output rainfall time-series is modelled as an $(n-m)^{\text{th}}$ order Integrated Random Walk (IRW) process described in the following section.

2.1. Estimation and implementation of regularised derivatives (RegDer method)

The use of regularised derivatives in model estimation is not a new development – Jakeman and Young (1984) show how recursive Kalman Filter (KF) algorithms (Kalman, 1960) and Fixed Interval Smoothing (FIS, e.g., Norton, 2009) produce reliable estimates of derivatives of time-series. Finite difference numerical schemes normally involve forms of direct differencing of signals, and so, while many will be stable, they will amplify the high frequency components of the discharge signal, thus producing noise artefacts. When they form filters with a degree of smoothing, they introduce filter artefacts, i.e., side lobes (FIR or polynomial filters effectively using combined central differences). Representative examples of this approach can be found *i.a.* in Luo et al. (2005), where the complicated spectra of Savitzky–Golay differentiators are shown. Other approaches to non-parametric derivative estimation (parametric estimation is seen as constraining) often involve forms of approximation in suitable functional bases including splines and other kernel smoothing forms. Derivative estimation or approximation is the subject of many studies e.g., De Brabanter et al. (2011), who use the kernel approach within a more complicated framework. Regularisation based derivative estimation was introduced several decades ago (Anderssen and Bloomfield, 1974). Most regularisation approaches use matrix-based methods involving operations on large matrices of the size of the data series, which is not practical for the long, frequently-sampled series used in hydrology and other environmental applications, unlike the recursive approach implemented here. Moussaoui et al. (2005) evaluated the possibilities of estimating derivatives and inputs of dynamic systems using regularisation techniques by applying a Tikhonov regularisation and then using Poisson filtering to jointly estimate parameters and signals. Their use of filtering techniques resulted in issues arising from phase lags in the estimated signals. They referred to Jakeman and Young (1984) with respect to possible solutions involving smoothing, but without proposing a method. In any case, smoothing is only applicable when rainfall is present at all times, which is not the case that this method is being developed to address.

As the rainfall and streamflow data are normally of time series nature with a fixed sampling rate, a discrete-time State-Space approach is employed to estimate the derivatives. This can be done because values between the sampling time instances are not used, and there is a direct equivalence between continuous-time and discrete-time models in regularly sampled data.

A basic discrete time Stochastic State-Space formulation is used (see e.g., Young et al., 1999) with the state transition equation as in Jakeman and Young (1984):

$$\boldsymbol{x}_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_k \tag{7}$$

where the state $\boldsymbol{x}_k = [Q_k \quad dQ_k]^T$ is composed of level state Q_k and slope state dQ_k of the Integrated Random Walk process which is used to describe $Q(t)$ with $t = k\Delta t$ where Δt is the sampling interval. It is this second component of the state dQ_k that provides the estimated time derivative of the observed process (given Δt). It is assumed that the discrete time is sampled uniformly with samples every time unit. The assumption is based on the fact that stage (and hence streamflow) is normally sampled uniformly by dataloggers. Rainfall is sampled normally using tipping-bucket raingauges and converted onto the same time basis as the streamflow data. The process is not observed directly, but through the observation equation:

$$Q_k^{obs} = [1 \quad 0]\boldsymbol{x}_k + e_k \tag{8}$$

where e_k and v_k are zero-mean, serially uncorrelated white noise sequences.

Equation (7) shows the manner of obtaining the 1st order derivative estimate, but it is easy to build up the State-Space to generate estimates of higher order derivatives.

The ratio of variances of the state- and observation-disturbance is termed the Noise Variance Ratio (NVR):

$$NVR = \frac{\sigma_v^2}{\sigma_e^2} \tag{9}$$

which is related, reciprocally, to the smoothness of the estimate, or the regularisation parameter (Jakeman and Young, 1984). This form of Stochastic State-Space formulation lends itself to the state estimation procedures of the KF and FIS (Bryson and Ho, 1969), noting that the combined KF/FIS algorithms produce not only optimal smooth estimates of both states but also estimates of their uncertainty bounds. The variance parameters σ_v^2 and σ_e^2, or in this simplified case the *NVR* parameter of the KF/FIS algorithm, are normally estimated using optimisation, usually involving Maximum Likelihood (ML) objective functions. Variants of the objective function are discussed by Tych et al. (2002) and Taylor et al. (2007). In the proposed approach, the objective function is modified from the usual ML approach to a measure of how well the estimated rainfall fits the actual rainfall time series. As the method is based primarily upon the use of Regularised Derivatives it is further called the *RegDer* method.

2.2. Comparison with the discrete-time inversion procedure (*InvTF* method)

For comparison with the *RegDer* method, the method of Andrews et al. (2010) based on the use of the direct inverse of a discrete transfer function, was also applied to the two catchment data sets. Since a discrete TF is used, the inverse is easy to simulate directly by differencing or near-differencing (i.e., no explicit differentiation). In discrete time form, this gives:

$$Q_k = \frac{\beta_0 + \beta_1 z^{-1} + \cdots + \beta_m z^{-m}}{1 + \alpha_1 z^{-1} + \cdots + \alpha_n z^{-n}} R_{k-\delta} \tag{10}$$

where the backward shift operator $z^{-1}y(k) = y(k-1)$ and $t = k\Delta t$ is the sample time of the kth sample. The operator z is used here instead of q (often used in system identification literature) to avoid confusion with standard hydrological practice that uses letter q to denote streamflow. The same notation and model orders were used for the parameters vector as for the CT-TF model (Equation (1)). Estimation of the discrete model was undertaken using the discrete version of the RIV method, implemented in the CAPTAIN Toolbox. The estimated rainfall time-series was then obtained simply by rearranging the above equation, as in Andrews et al. (2010):

$$R_{k-1} = \frac{1}{\beta_0}\{(Q_k + \alpha_1 Q_{k-1} + \alpha_2 Q_{k-2}) - (\beta_1 R_{k-2} + \beta_2 R_{k-3})\} \tag{11}$$

This is shown here for $n = m = 2$ and $\delta = 1$ for clarity. As with the continuous-time form, the time delay, estimated from the data, can be removed during the off-line processing. This approach, based on a direct inverse of a discrete transfer function (Andrews et al., 2010), is here called *InvTF*.

3. First evaluation of the new *RegDer* methodology (including *InvTF* comparisons)

In order to evaluate the *RegDer* algorithm's performance, data from two headwater experimental catchments exhibiting both contrasting rainfall regimes and hydrological pathways were compared. Previous studies have identified linear models for both catchments (Chappell et al., 2006 – Baru; Ockenden and Chappell, 2011 – Blind Beck). Subsequent analysis using the classic bilinear power law (Beven, 2011) has confirmed this assumption. On this basis, linear modelling was applied in both cases. Streamflow was sampled uniformly by dataloggers, while rainfall was sampled using tipping-bucket raingauges then converted onto the same time basis as the streamflow data.

3.1. Choice of evaluation metrics

Alexandrov et al. (2011) suggest a general framework for model assessment and a wide variety of possible metrics are available. Bennett et al. (2013) present a range of possible tests including numerical, graphical and qualitative techniques and a selection of these was employed in this study. Some were found to be inappropriate as they involve a normal distribution of data and/or residuals or other critical assumptions. Q–Q plots of the residuals (not shown here) clearly indicated that the assumption of normality cannot be made. In future work, decision theory may provide a framework for choosing between both modelling methods and competing model structures.

Commonly, the simplified Nash–Sutcliffe Efficiency (NSE or R_t^2) is used to compare the performance of hydrological models. Several models may be identified which fit the data well (i.e., equifinality: Beven, 2006) so the Young Information Criterion (YIC: Young, 2001) can be used to differentiate between these models. The YIC is an objective measure combining the goodness of fit with a measure of over-parameterisation.

Once acceptable forward models (i.e., rainfall–runoff) have been selected (using R_t^2 and *YIC*) they are inverted and the performance of the inverse models compared using a range of metrics including R_t^2, basic statistics of the residuals and visual ability to match peak values. The inferred (or synthetic) rainfall sequences were also

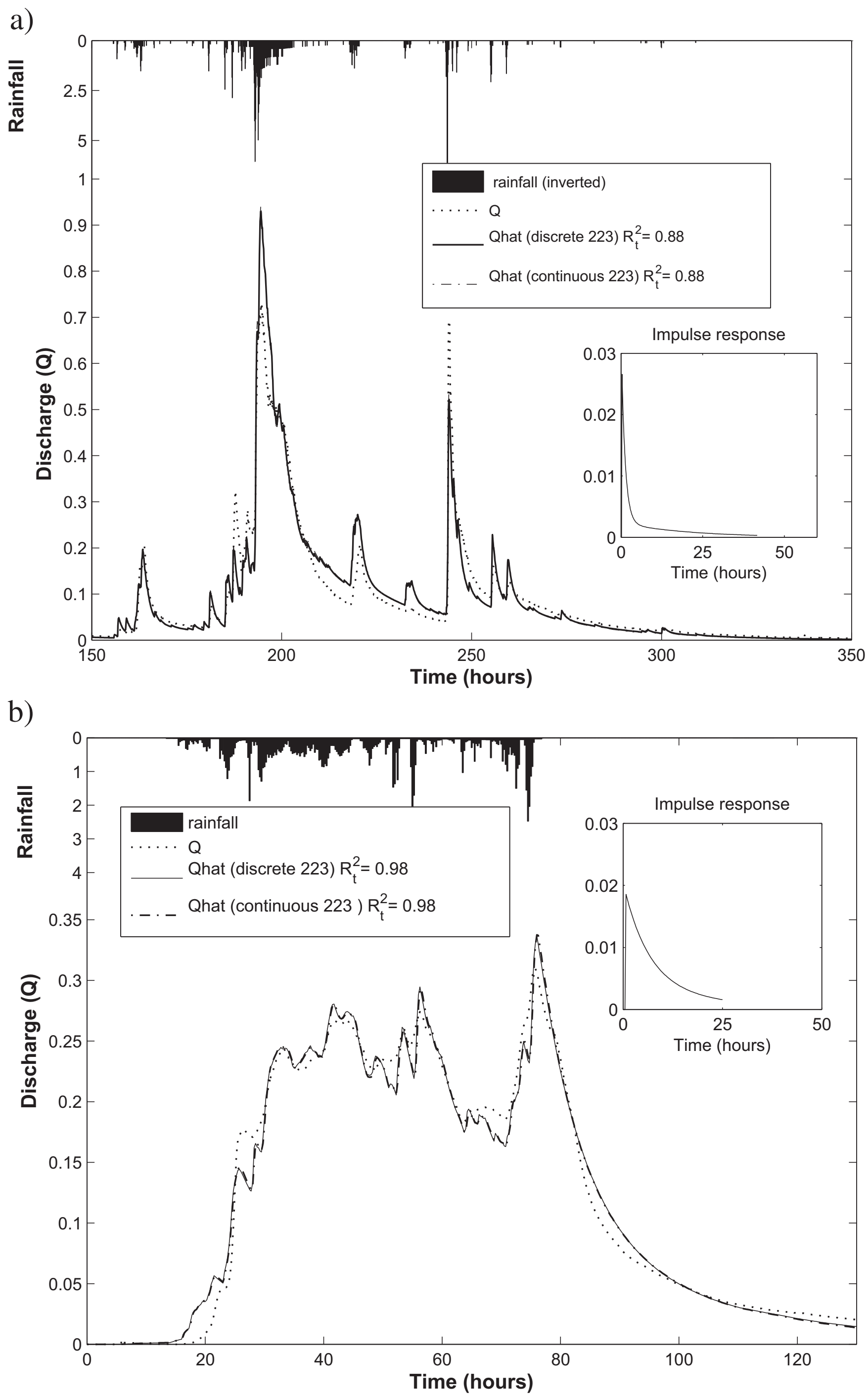

Fig. 2. Measured and estimated streamflow for: a) Baru (at 5 min intervals) and b) Blind Beck (at 15 min intervals), together with the associated hyetograms and impulse responses.

compared visually with each other and with the observed rainfall. Inferred and observed rainfall series were then used as inputs to the original forward models and the generated modelled flow sequences compared using the R_t^2 values and visual comparisons. Statistical analysis of the residuals of both models gives an additional insight into the differences between the catchments and rainfall regimes, as well as the differences between the inversion approaches.

Model uncertainty is evaluated using Monte Carlo Simulations (MCS) for both the forward and the inverse models utilising the covariance matrix generated as part of the output from the estimation routines contained in the CAPTAIN Toolbox for Matlab (Taylor et al., 2007). In this analysis, the guidelines for validation of DBM models published by Young (2001) are followed. The models thus generated can be used to investigate the sensitivity of the inversion process to the parameterisation of the forward model.

3.2. Data: Baru tropical catchment responses

The 0.44 km^2 Baru catchment is situated in the headwaters of the Segama river located in Sabah on the northern tip of Borneo, East Malaysia (4° 58′ N 117° 49′ E). The climate is equatorial with a

Table 1
The best CT-TF models fitted to subsets of data for Blind Beck (sampled at 15 min intervals) and Baru (sampled at 5 min intervals). There is little difference in efficiency (R_t^2) between the different models so selection was based on the lowest order model with the lowest YIC (Young, 2001). The YIC is an objective measure combining the goodness of fit with a measure of over-parameterisation. A model with a large negative YIC fits the data well with a small number of parameters.

Catchment	Model structure [n,m,d]	R_t^2	YIC	Time constants (h)	
				1st	2nd
Blind Beck	[2,2,3]	0.983	−6.711	6.35	22.10
Baru	[2,2,3]	0.878	−8.054	1.14	20.56

twenty-six year (1985–2010) mean rainfall of 2849 mm (Walsh et al., 2011) showing no marked seasonality but tending to fall in short (<15 min) convective events showing high spatial variability and intensities much higher than those of temperate UK (Bidin and Chappell, 2003, 2006). Due to the high spatial variability, a network of 6 automatic rain-gauges (13.6 gauges per km^2) was used to derive the catchment-average rainfall using the Thiessen Polygon method. Haplic alisols, typically 1.5 m in depth and with a high infiltration capacity (Chappell et al., 1998) are underlain by relatively impermeable mudstone bedrock resulting in the dominance of comparatively shallow sub-surface pathways in this basin (Chappell et al., 2006). As a result of the high rainfall intensity and shallow water pathways the stream response is very flashy (i.e., rapid recession in the impulse response function). The data used in the analysis are from February 1996 sampled at 5 min intervals (Fig. 2a) and have been modelled previously by Chappell et al. (1999) and Walsh et al. (2011).

3.3. Data: Blind Beck temperate catchment response

The Blind Beck catchment has an area of 8.8 km^2 and lies in the headwaters of the Eden basin in North West England, UK (54.51°N 2.38°W). The basin's response shows evidence of deep hydrological pathways due to the presence of deep limestone and sandstone aquifers, and this has resulted in a damped hydrograph response (Mayes et al., 2006; Ockenden and Chappell, 2011; Ockenden et al., 2014). Winter rainfall in this basin is derived from frontal systems with typically lower intensities than the convective systems in the tropics (Reynard and Stewart, 1993). Data from a single tipping-bucket raingauge (i.e., 0.1 gauges per km^2) located in the middle of the catchment was used in this study. The data used in the analysis covers the period from 26th Dec 2007 at 16:45 to 31st December 2007 at 21:45 sampled at 15 min intervals (Fig. 2b) and was previously modelled by Ockenden and Chappell (2011).

The choice of these two experimental catchments, therefore, allowed the initial evaluation of the estimation of catchment rainfall from streamflow for the end-member extremes of a basin with tropical convective rainfall and shallow flow pathways to a basin with temperate frontal rainfall (i.e., much lower intensity) and deep flow pathways (i.e., much greater basin damping or temporal integration).

4. First results and discussion

Forward CT-TF models identified for Blind Beck data explained over 98% of the variance in the streamflow, whilst those for the Baru fit slightly less well, explaining 88% – see Table 1 for the R_t^2, YIC (Young, 2001), and time-constants of the best forward models for each catchment, based on a high R_t^2 with a large negative YIC value according to DBM methodology. The simulated streamflows from a 2nd-order model for the two basins are shown in Fig. 2. The impulse response function (i.e., unit hydrograph) for the Baru catchment (Fig. 2b) showed a considerably faster recession in comparison to that of the Blind Beck catchment (Fig. 2a) by a factor of 6, confirming the more flashy nature of the shallow, tropical catchment, as noted by previous transfer function studies (Chappell et al., 1999, 2006, 2012; Walsh et al., 2011).

The identified well-fitting, forward models selected according to the DBM methodology were then inverted using the *RegDer* method and, for comparison, the *InvTF* method to estimate catchment rainfall from streamflow for the two catchments. The results of the inversions using the two techniques are shown in Fig. 3 and the reverse models' fit in Table 2.

Both approaches applied to the streamflow data for the Blind Beck catchment produced very similar inferred rainfall time-series (Fig. 3b). Both approaches produce slightly smoothed rainfall time-series compared to the observed 15-min sampled rainfall. The smoothing effect is small when compared with the time constant of 6.4 h for the main component of the forward CT-TF model for the Blind Beck catchment (Table 1). Both produce some briefly negative rainfall values during periods of hydrograph recession. Estimated periods of negative rainfall are likely to be due to the point (i.e., highly localised) rainfall measurements not fully characterising the entire catchment rainfall, so, at times, there is discharge with no locally measured rainfall that could be attributed to it, and *vice-versa*; an effect also described by Young and Sumisławska (2012).

In general, the forward models fit very well so the uncertainty bounds demonstrated by Monte Carlo runs are very narrow as illustrated in Fig. 3.

When applied to the Baru data, the *RegDer* and *InvTF* approaches do, however, give simulated or synthetic rainfall time-series with some different characteristics (Fig. 3a). The *InvTF* method, while capturing some of the peaks better (illustrated in Fig. 4 and Table 3) gives a time-series with very high frequency noise component, of such a high intensity that it produces momentary negative rainfall values. These very high frequency components are the result of the direct differencing involved in this method of inversion, which severely amplifies high frequency noise in the signal. In contrast, the *RegDer* method again produced smoothed inferred rainfall time-series with dynamics faster than the time constant of 1.14 h for the faster component of the forward CT-TF model for the Baru catchment (Table 1). An interesting insight is gained by examining the inset in Fig. 3b, where the two inferred rainfall series clearly follow the same trajectory, but the *InvTF* results include the high frequency noise, very clearly not related to the observed rainfall. The observed rainfall is indeed smoother than its *InvTF* estimate. These artefacts manifest themselves to a much higher degree in the fast responding Baru catchment with a different rainfall regime.

This last observation is confirmed by the residuals analysis. Residuals plots are shown in Fig. 3a and b for Baru and Blind Beck respectively. It is apparent from the plots how much more high frequency noise is involved in the *InvTF* estimates, even for the Blind Beck data, where both methods perform in a similar manner (see the residuals variance values in the plots). Fig. 5 shows comparative plots of the residuals autocorrelation function (RACF) for both models and both catchments. As expected the RACFs for Blind Beck are similar, quickly disappearing within their confidence bounds and it is just the variance level that differentiates the results for both methods. For Baru the RACFs are quite different, with RACF for *RegDer* quickly attenuated and not showing the negative ACF values characterising the fast switching, noisy *InvTF* residuals.

Table 3 shows that while the residuals statistics for Blind Beck show good similarity between the methods, the residuals for Baru show large discrepancies, with *InvTF* showing some extreme values and a completely different distribution shape, as characterised by the calculated moments: means are similar, variance doubles for *InvTF*, and higher moments are radically different and not realistic.

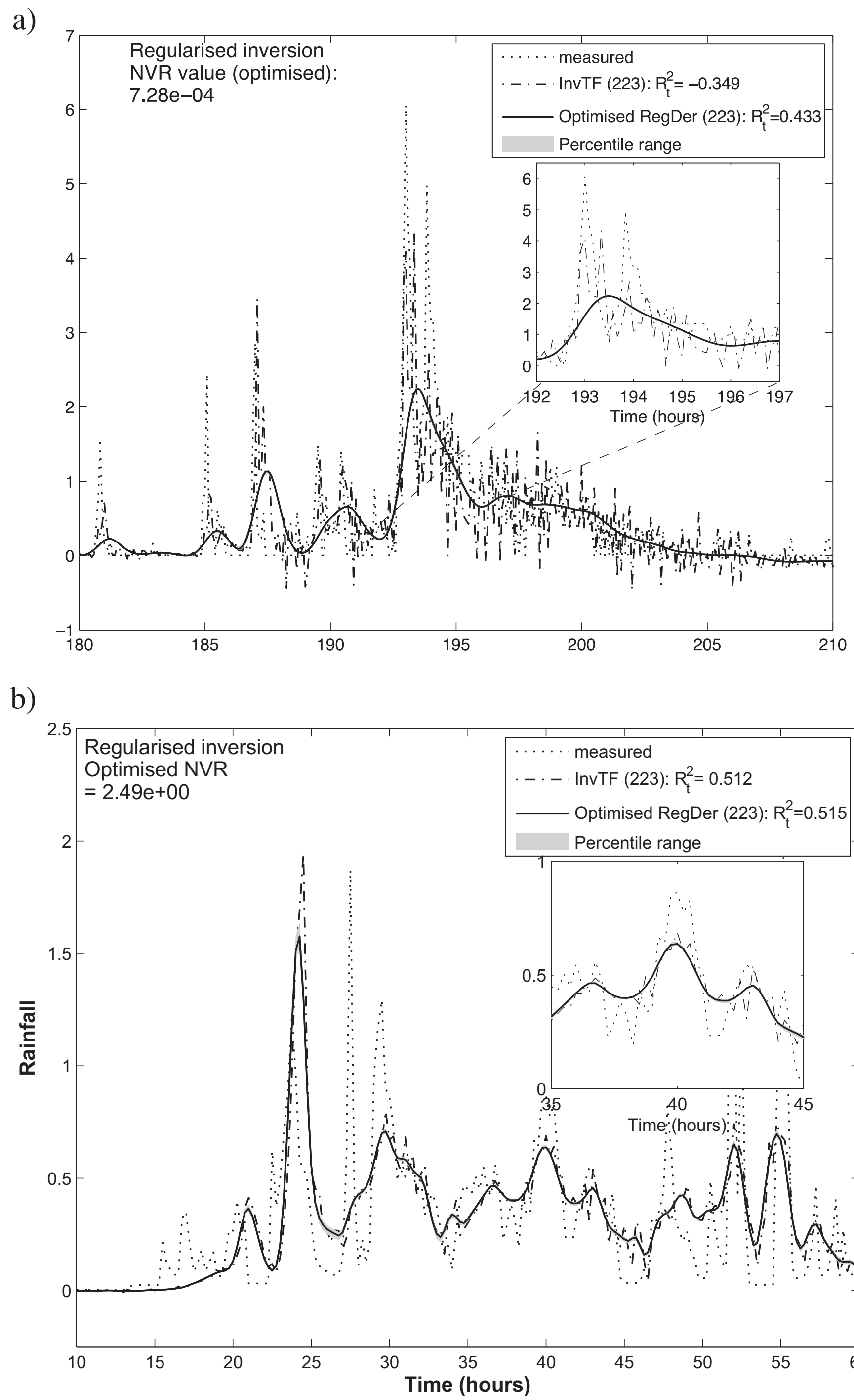

Fig. 3. Comparison of rainfall simulated using the *InvTF* and *RegDer* (NVR optimised) methods for a) Baru and b) Blind Beck. Examination of the inset confirms that the *RegDer* method estimates the Baru catchment rainfall better (see Table 2) whilst there is little difference between the methods for Blind Beck rainfall. 99% uncertainty bands generated by Monte Carlo analysis are shown and can be seen to be very narrow.

The Mean Absolute Error statistics (MAE) show similar relationships to the variance.

Similar effects are shown by the peaks statistics (Bennett et al., 2013) in Fig. 6. In the figure P_e denotes effective rainfall, while P_{eh} – inferred effective rainfall. The errors in peak estimates are of similar magnitude. Inferred in this figure refers to the values of peaks of inferred rainfall. Baru results show considerable improvement of these peak error statistics achieved using *RegDer* approach.

Despite the presence of smoothing effects and/or high frequency noise components, models simulating observed streamflow from synthetic rainfall using either method were able to simulate the observed streamflow equally well, and with a very high efficiency (Table 4), resulting in virtually indistinguishable model outputs given the observed rainfall or *RegDer* or *InvTF* rainfall as inputs. This is demonstrated in Fig. 7a and b.

It should be noted that while *RegDer* results appear to be 'too smooth' and the *InvTF* results – too 'noisy', the balance between the two is easily achieved using *RegDer* by balancing the NVR coefficients of the inverse model, and will ultimately be up to the researcher and the aims of modelling exercise. *RegDer* results can

Table 2
Efficiency (R_t^2) values for the rainfall sequences estimated by inverting the models selected for Blind Beck and Baru using the *InvTF* and *RegDer* methods of inversion.

R_t^2	Blind Beck [2,2,3]	Baru [2,2,3]
InvTF	0.512	−0.349
RegDer	0.515	0.433

be interpreted as sub-sampling, or sacrificing the unobtainable (due to observation disturbance) temporal resolution. Critically, there are no such controls with *InvTF*. Quantifying this balance is a part of on-going research and is to be addressed in a forthcoming publication. Applying a smoothing algorithm to *InvTF* results would produce a different outcome, as *RegDer* only applies regularisation to the minimal number of terms within the bank of filters of

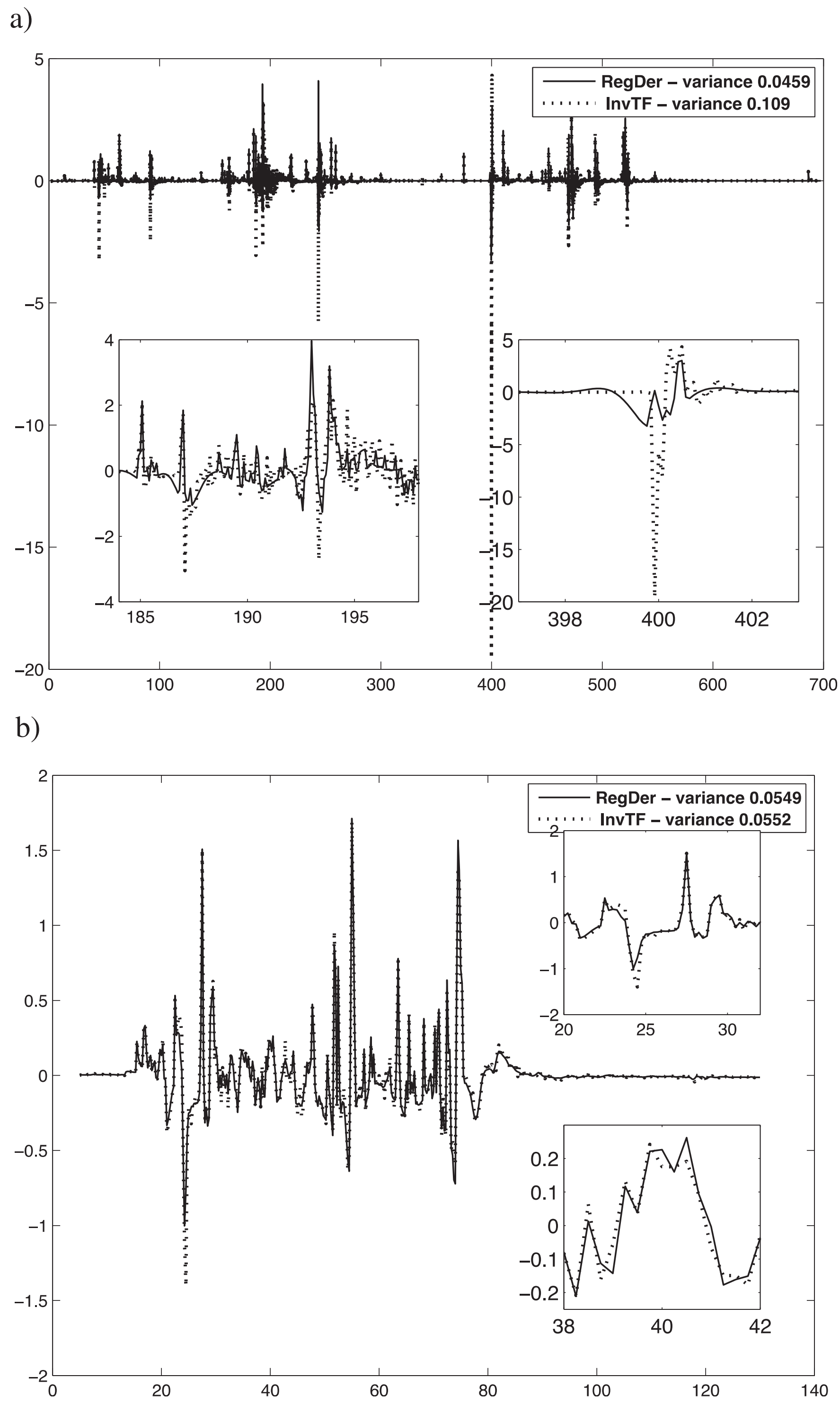

Fig. 4. Comparison of residuals for a) Baru and b) Blind Beck for the two inversion methods showing the similarities in performance between the methods when used for Blind Beck (with a minor increase in noise for *InvTF*) and the differences when used for Baru (with large artefacts in *InvTF*).

Table 3
Residuals analysis for Blind Beck and Baru for both inversion methods showing the similarity between the methods for Blind Beck and the differences for Baru.

	Mean	Mode	Var	Skew	Kurt	Max	Min	Rng	MAE
Blind Beck									
RegDer	−0.0004	−0.0119	0.0549	2.54	20.1	1.71	−1.00	2.71	0.117
InvTF	0.0001	−1.4012	0.0552	1.77	19.4	1.69	−1.40	3.09	0.118
Baru									
RegDer	−0.004	0.0001	0.0459	3.51	112.3	4.09	−3.25	7.34	0.057
InvTF	−0.0036	0	0.1092	−27.17	1549.2	4.31	−19.56	23.9	0.066

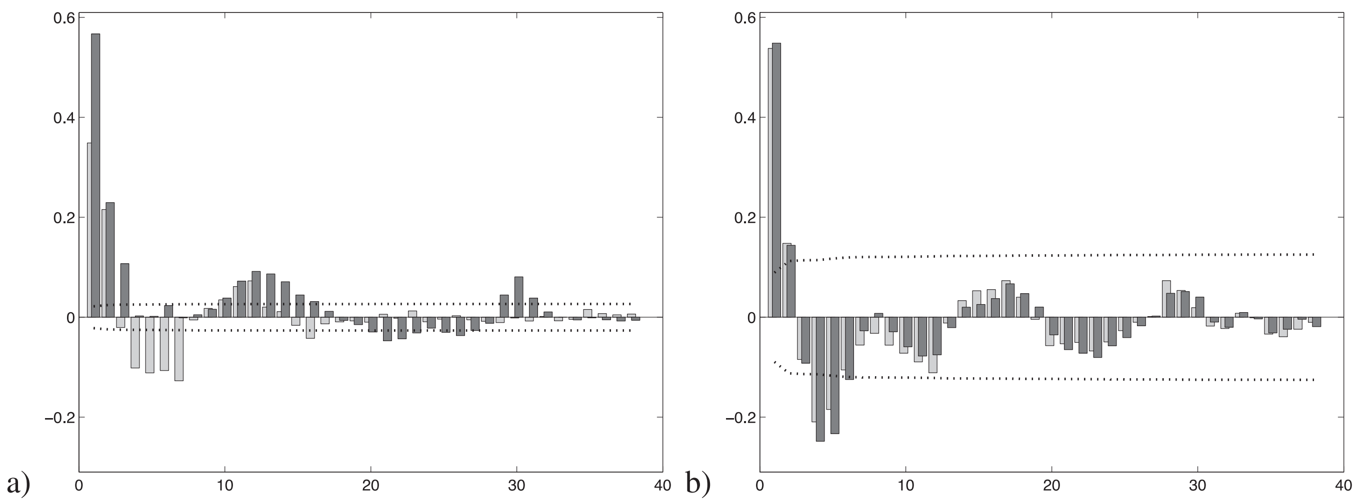

Fig. 5. Comparative plots of the residuals autocorrelation function (RACF) for *InvTF* (light grey bars) and *RegDer* (dark grey bars) and both catchments (Baru in (a) and Blind Beck in (b)) showing the differences between methods of inversion. In both cases, *RegDer* quickly attenuates whereas *InvTF* shows negative ACF values characterising the fast switching, noisy residuals/artefacts.

Equation (5), as opposed to a cruder tool of smoothing the entire signal.

The integrating effect of the Blind Beck catchment seen in the damped hydrograph (Fig. 7b) was expected given the presence of deeper hydrological pathways (Ockenden and Chappell, 2011; Ockenden et al., 2014) however, the degree of temporal basin integration of the rainfall signal (and hence response damping) by the shallow pathways within the tropical catchment (Chappell et al., 2006) was not expected, but does indicate the role of even shallow water paths in damping intense rainfall. The degree of

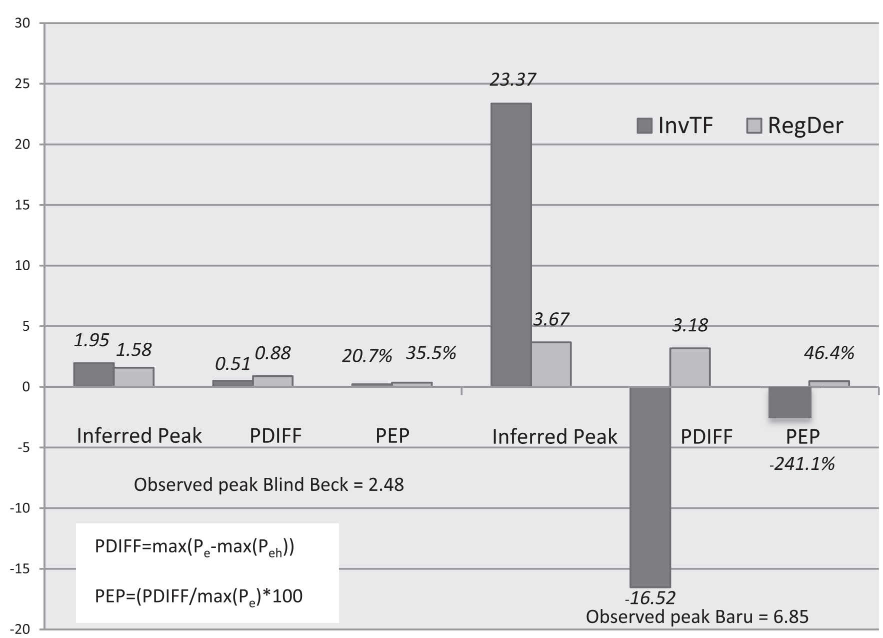

Fig. 6. Comparison of the estimation of peaks for the two methods showing that for Blind Beck, both methods estimate the observed peak quite well with little difference between them whilst for Baru, the *InvTF* method hugely underestimates the peak whilst *RegDer* slightly over-estimates. The metrics PDIFF and PEP were taken from Bennett et al. (2013).

Table 4
Efficiency (R_t^2) of forward CT-TF models of streamflow based on the observed rainfall or *RegDer* or *InvTF* rainfall as inputs.

Model input	Blind Beck R_t^2	Baru R_t^2
Observed rain	0.984	0.878
Modelled rain (*InvTF*)	1.000	0.937
Modelled rain (*RegDer*)	1.000	0.957

catchment integration indicates that the slight smoothing of the simulated rainfall time-series (by the *RegDer* method) has no impact on its ability to be used in forward CT-TF models to simulate streamflow. On the basis of their utility for creating synthetic rainfall time-series for use in periods lacking observed rainfall, the new *RegDer* method and *InvTF* method of Andrews et al. (2010) seem of equal value. Perhaps the new *RegDer* method is marginally better than the *InvTF* method because of the high frequency behaviour that can be produced by the *InvTF* method with some data sets where high frequency noise is amplified by the derivative action, for example, the proposed approach is more robust for stiff systems (those with a wide range of time constants). Further, this high frequency behaviour has no physical interpretation so might be considered to fail the final evaluation criterion of the DBM modelling philosophy (Chappell et al., 2012). These findings from the first evaluation of the new *RegDer* method are very positive and

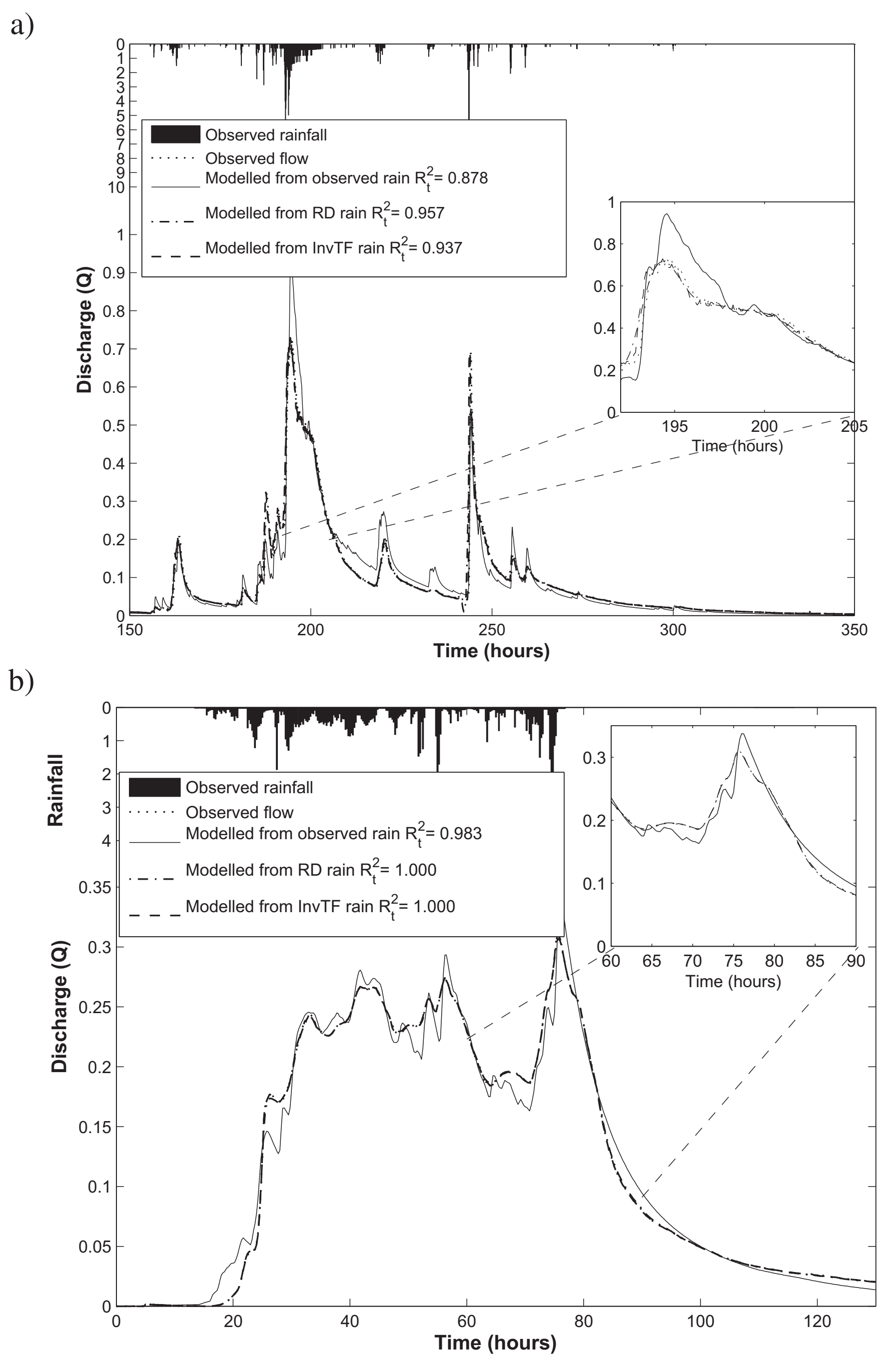

Fig. 7. Outputs modelled from observed and modelled rainfall sequences for a) Baru and b) Blind Beck showing that the outputs (discharges) are indistinguishable over much of the figure despite the differing characteristics of the rainfall inputs.

Table 5
Data and model output statistics (rainfall). The following abbreviations were used: Var – variance, Kurt – kurtosis, Skew – skewness, IQR – inter-quartile range, Prct – percentiles. Obs refers to observed rainfall. The *Wet* prefix in the table rows refers to statistics calculated only for samples with non-zero rainfall (>0 for inferred).

Blind Beck	Mean	Var	Skew	Kurt	Max	Min	Range	25% Prct	75% Prct	IQR
Obs. all	0.181	0.112	3.154	15.934	2.476	0.003	2.474	0.004	0.233	0.230
Obs. wet	0.181	0.112	3.152	15.925	2.476	0.000	2.476	0.004	0.233	0.230
RegDer	0.182	0.061	1.744	7.451	1.576	−0.156	1.733	0.010	0.319	0.309
InvTF	0.181	0.067	2.120	10.591	1.948	−0.198	2.146	0.008	0.311	0.303
Wet *RegDer*	0.202	0.062	1.658	7.289	1.576	−0.129	1.705	0.012	0.342	0.330
Wet *InvTF*	0.198	0.069	2.065	10.581	1.948	−0.198	2.146	0.010	0.345	0.335
Baru										
Obs. all	0.050	0.081	11.230	179.694	6.853	0.000	6.853	0.000	0.000	0.000
Obs. wet	0.253	0.403	4.383	27.969	6.056	0.000	6.056	0.000	0.213	0.213
RegDer	0.054	0.054	7.549	76.584	3.674	−0.392	4.066	0.001	0.018	0.017
InvTF	0.054	0.169	29.739	1411.93	23.374	−3.630	27.004	0.001	0.018	0.018
Wet *RegDer*	0.055	0.042	6.751	60.481	2.763	−0.336	3.099	0.001	0.020	0.019
Wet *InvTF*	0.051	0.095	18.517	567.320	12.644	−1.461	27.004	0.001	0.017	0.017

highlight the potential value of this method for generating synthetic rainfall time-series for a range of rainfall regimes and catchment settings. These preliminary findings have stimulated a much more extensive programme of evaluation of the *RegDer* method against a range of other methods (including the *InvTF* method of Andrews et al., 2010) for a much larger set of catchments with differing rainfall and catchment settings.

A number of basic statistics of the observed and inferred (*RegDer* and *InvTF*) rainfall series are shown in Table 5. It is clear that for the Blind Beck catchment most statistics for both observed and inferred series are similar in magnitude (they were not expected to be too close due to the smoothing effect of both methods), which is consistent with other results reported above. For Baru however, there are significant differences between the methods. There is an indication of mean-smoothing effects of both methods showing in variance and range. *InvTF* inferred rainfall shows large changes and unusual values in range, minima and maxima, as well as higher order moments being of different order of magnitude from those of the actual rainfall and *RegDer* results. This is an indication of the artefacts of explicit differencing of the streamflow data when using *InvTF*. In addition the high skewness of the observed rainfall measurements adds to the argument regarding non-Gaussian distribution, and hence many of the standard model metrics not being applicable.

5. Conclusions

Robust identification techniques were used to identify continuous-time transfer function models for two catchments with contrasting rainfall and flow path regimes. Following the DBM methodology, the models fitted the data well with a minimal number of parameters as indicated by a large negative value of the YIC. The identified (DBM) models for both catchments were of 2nd-order. This is a typical model order for many catchments. The models were inverted using the new *RegDer* method and, for comparison, the *InvTF* method used by Andrews et al. (2010). Both methods were able to produce synthetic rainfall time-series that were then able to simulate almost all of the dynamics in the streamflow time-series for both catchments (Fig. 4a, b). In comparison to the *InvTF* method of Andrews et al. (2010), the *RegDer* method did, however, produce synthetic rainfall containing much less high frequency noise. This was particularly visible in the synthetic rainfall of *InvTF* for the tropical basin with convective rainfall (Fig. 3a). The smoothing introduced by the *RegDer* method is on a much smaller temporal scale than the dominant dynamics of the catchment indicating that the detailed temporal distribution of the rainfall series may not be important for the modelling the observed streamflow (depending on the reasons for modelling) so long as the series recreates the short-term (i.e., sub-hourly) characteristics responsible for producing stream hydrographs sufficiently well, which is consistent with the findings of Eagleson (1967) and Obled et al. (1994). These findings are confirmed by comparative evaluation of several model metrics, including peak modelling errors and a detailed residuals analysis. It is worth noting that applying a smoothing algorithm to *InvTF* results would produce a different outcome, as *RegDer* only applies regularisation to the minimal number of terms within the bank of filters of Equation (5), as opposed to a cruder tool of smoothing the entire signal.

Further evaluations of the new *RegDer* method against *InvTF* and other methods need to be undertaken using a more diverse range of global rainfall and flow-path regimes. This work will include catchments where the derivation of long-term rainfall time-series by *RegDer* would support hydrological, climatological or ecological studies requiring such long time-series of synthesised rainfall (Ormerod and Durance, 2009).

Software and data used to produce the results in this paper are available upon request from the corresponding author.

Acknowledgements

The authors would like to thank Mary Ockenden for the collection and quality assurance of the period of rainfall and streamflow for the Blind Beck catchment (NERC grant number NER/S/A/2006/14326), and also Jamal Mohd Hanapi and Johnny Larenus for the collection of the period of rainfall and streamflow utilised for the Baru catchment and to Paul McKenna for its quality assurance (NERC grant number GR3/9439). This work has been partly supported by the Natural Environment Research Council [Consortium on Risk in the Environment: Diagnostics, Integration, Benchmarking, Learning and Elicitation (CREDIBLE)] grant number: NE/J017299/1.

References

Alexandrov, G.A., Ames, D., Bellocchi, G., Bruen, M., Crout, N., Erechtchoukova, M., Hildebrandt, A., Hoffman, F., Jackisch, C., Khaiter, P., Mannina, G., Matsunaga, T., Purucker, S.T., Rivington, M., Samaniego, L., 2011. Technical assessment and evaluation of environmental models and software: letter to the editor. Environ. Model. Softw. 26 (3), 328–336.

Anderssen, R., Bloomfield, P., 1974. Numerical differentiation procedures for non-exact data. Numer. Math. 22, 157–182.

Andrews, F., Croke, B., Jeanes, K., 2010. Robust estimation of the total unit hydrograph. In: 2010 International Congress on Environmental Modelling and Software Modelling for Environment's Sake. Ottawa, Canada.

Andrews, F., Croke, B., Jakeman, A., 2011. An open software environment for hydrological model assessment and development. Environ. Model. Softw. 26 (10), 1171–1185.
Antsaklis, P., 1978. Stable proper nth-order inverses. IEEE Trans. Autom. Control 23 (6), 1104–1106.
Bardossy, A., Das, T., 2008. Influence of rainfall observation network on model calibration and application. Hydrol. Earth Syst. Sci. 12 (1), 77–89.
Bennett, N.D., Croke, B.F.W., Guariso, G., Guillaume, J.H.A., Hamilton, S.H., Jakeman, A.J., Marsili-Libelli S,Newham, L.T.H., Norton, J.P., Perrin, C., Pierce, S.A., Robson, B., Seppelt, R., Voinov, A.A., Fath, B.D., Andreassian, V., 2013. Characterising performance of environmental models. Environ. Softw. 40, 1–20.
Beven, K., 2006. A manifesto for the equifinality thesis. J. Hydrol. 320, 18–36.
Beven, Keith J., 2011. Rainfall–Runoff Modelling – the Primer, second ed. John Wiley and Sons, Chichester, England.
Beven, K., Smith, P., 2014. Concepts of information content and likelihood in parameter calibration for hydrological simulation models. J. Hydrol. Eng. 10, 1061.
Bidin, K., Chappell, N., 2003. First evidence of a structured and dynamic spatial pattern of rainfall within a small humid tropical catchment. Hydrol. Earth Syst. Sci. 7 (2), 245–253.
Bidin, K., Chappell, N., 2006. Characteristics of rain events at an inland locality in Northeastern Borneo, Malaysia. Hydrol. Process. 20 (18), 3835–3850.
Box, G.E.P., Jenkins, G.M., Reinsel, G.C., 2008. Time Series Analysis: Forecasting and Control, fourth ed.
Bryson, A.E., Ho, Y.-C., 1969. Applied Optimal Control: Optimization, Estimation, and Control.
Chappell, N., Franks, S., Larenus, J., 1998. Multi-scale permeability estimation for a tropical catchment. Hydrol. Process. 12 (9), 1507–1523.
Chappell, N.A., Mckenna, P., Bidin, K., Douglas, I., Walsh, R.P.D., 1999. Parsimonious modelling of water and suspended sediment flux from nested catchments affected by selective tropical forestry. Philos. Trans. R. Soc. Lond. Ser. B Biol. Sci. 354 (1391), 1831–1846.
Chappell, N.A., Tych, W., Chotai, A., Bidin, K., Sinun, W., Chiew, T.H., 2006. Barumodel: combined data based mechanistic models of runoff response in a managed rainforest catchment. For. Ecol. Manag. 224 (1), 58–80.
Chappell, N.A., Bonell, M., Barnes, C., Tych, W., 2012. Tropical cyclone effects on rapid runoff responses: quantifying with new continuous-time transfer function models. In: Webb, A.A., Bonell, M., Bren, L., Lane, P.N.J., Mcguire, D., Neary, D.G., Nettles, J., Scott, D.F., Stednick, J., Wang, Y. (Eds.), Revisiting Experimental Catchment Studies in Forest Hydrology: Proceedings of a Workshop Held during the XXV IUGG General Assembly in Melbourne. June–July 2011. International Association of Hydrological Sciences (IAHS), Wallingford.
Croke, B., 2006. A technique for deriving an average event unit hydrograph from streamflow – only data for ephemeral quick-flow-dominant catchments. Adv. Water Resour. 29 (4), 493–502.
Cunha, L., Mandapaka, P., Krajewski, W., Mantilla, R., Bradley, A., 2012. Impact of radar-rainfall error structure on estimated flood magnitude across scales: an investigation based on a parsimonious distributed hydrological model. Water Resour. Res. 48.
De Brabanter, K., De Brabanter, J., De Moor, B., 2011. Nonparametric derivative estimation. In: Proceedings of the 23rd Benelux Conference on Artificial Intelligence.
Eagleson, P., 1967. Optimum density of rainfall networks. Water Resour. Res. 3 (4), 1021.
Fujihara, Y., Simonovic, S., Topaloglu, F., Tanaka, K., Watanabe, T., 2008. An inverse-modelling approach to assess the impacts of climate change in the Seyhan River Basin, Turkey. Hydrol. Sci. J.-J. Des. Sci. Hydrol. 53 (6), 1121–1136.
Hino, M., 1986. Improvements in the inverse estimation of effective rainfall from runoff. J. Hydrol. 83 (1–2), 137–147.
Hjelmfelt, A., 1981. Overland flow from time-distributed rainfall. J. Hydraul. Div.-ASCE 107 (2), 227–238.
Jakeman, A., Young, P., May 1984. Recursive filtering and smoothing procedures for the inversion of ill-posed causal problems. Util. Math. 25, 351–376.
Kalman, R.E., 1960. A new approach to linear filtering and prediction problems. Trans. ASME J. Basic Eng. 82 (Series D), 35–45.
Kirchner, J., 2009. Catchments as simple dynamical systems: catchment characterization, rainfall–runoff modeling, and doing hydrology backward. Water Resour. Res. 45.
Krier, R., Matgen, P., Goergen, K., Pfister, L., Hoffmann, L., Kirchner, J.W., Uhlenbrook, S., Savenije, H.H.G., 2012. Inferring catchment precipitation by doing hydrology backward: a test in 24 small and mesoscale catchments in Luxembourg. Water Resour. Res. 48, W10525.
Littlewood, I., Croke, B., 2013. Effects of data time-step on the accuracy of calibrated rainfall-streamflow model parameters: practical aspects of uncertainty reduction. Hydrol. Res. 44 (3), 430–440.
Luo, Jianwen, Ying, Kui, Bai, Jing, 2005. Savitzky–Golay smoothing and differentiation filter for even number data. Signal Process. 85 (7), 1429–1434.
Maquin, 1994. Estimation of unknown inputs in linear systems. In: American Control Conference, ISBN 0-7803-1783-1, pp. 1195–1197.
Mayes, W.M., Walsh, C.L., Bathurst, J.C., Kilsby, C.G., Quinn, R.F., Wilkinson, M.E., Daugherty, A.J., Connell, P.E., 2006. Monitoring a flood event in a densely instrumented catchment, the Upper Eden, Cumbria, UK. Water Environ. J. 20 (4), 217–226.
Moussaoui, S., Brie, D., Richard, A., 2005. Regularization aspects in continuous-time model identification. Automatica 41 (2), 197–208.
Neumaier, A., 1998. Solving ill-conditioned and singular linear systems: a tutorial on regularization. SIAM Rev..
Norton, J.P., 2009. An Introduction to Identification. Dover Publications.
O'Sullivan, F., 1986. A statistical perspective on ill-posed inverse problems. Stat. Sci. 1 (4), 502–518.
Obled, C., Wendling, J., Beven, K., 1994. The sensitivity of hydrological models to spatial rainfall patterns – an evaluation using observed data. J. Hydrol. 159 (1–4), 305–333.
Ockenden, M.C., Chappell, N.A., 2011. Identification of the dominant runoff pathways from data-based mechanistic modelling of nested catchments in temperate UK. J. Hydrol. 402 (1), 71–79.
Ockenden, M.C., Chappell, N.A., Neal, C., 2014. Quantifying the differential contributions of deep groundwater to streamflow in nested basins, using both water quality characteristics and water balance. Hydrol. Res. 45, 200–212.
Ogden, F., Julien, P., 1994. Runoff model sensitivity to radar resolution. J. Hydrol. 158 (1–2), 1–18.
Ormerod, S., Durance, I., 2009. Restoration and recovery from acidification in upland Welsh streams over 25 years. J. Appl. Ecol. 46 (1), 164–174.
Reynard, N.S., Stewart, E.J., 1993. The derivation of design rainfall profiles for upland areas of the UK. Meteorol. Mag. 122, 116–123.
Sherman, L.K., 1932. Streamflow from rainfall by the unitgraph method. Eng. News Rec. 108, 501–505.
Tarantola, A., 2005. Inverse Problem Theory and Methods for Model Parameter Estimation: Society for Industrial and Applied Mathematics.
Taylor, C.J., Pedregal, D.J., Young, P.C., Tych, W., 2007. Environmental time series analysis and forecasting with the Captain toolbox. Environ. Model. Softw. 22 (6), 797–814.
Teuling, A.J., Lehner, I., Kirchner, J.W., Seneviratne, S.I., 2010. Catchments as simple dynamical systems: experience from a Swiss prealpine catchment. Water Resour. Res. 46 (10).
Tych, W., Pedregal, D.J., Young, P.C., Davies, J., 2002. An unobserved component model for multi-rate forecasting of telephone call demand: the design of a forecasting support system. Int. J. Forecast. 18 (4), 673–695.
Walsh, R.P.D., Bidin, K., Blake, W.H., Chappell, N.A., Clarke, M.A., Douglas, I., Ghazali, R., Sayer, A.M., Suhaimi, J., Tych, W., Annammala, K.V., 2011. Long-term responses of rainforest erosional systems at different spatial scales to selective logging and climatic change. Philos. Trans. R. Soc. Lond. 366, 3340–3353.
Wang, W., Henriksen, R., 1994. Generalized predictive control of nonlinear systems of the Hammerstein form. Model. Identification Control 15 (4), 253–262.
Yang, F., Wilde, R., 1988. Observer for linear systems with unknown inputs. IEEE Trans. Autom. Control 33 (7), 677–681.
Young, P.C., 1998. Data-based mechanistic modelling of environmental, ecological. Econ. Eng. Syst. 13 (2), 105–122.
Young, P.C., 1999. Data-based mechanistic modelling. Gen. Sensit. Domin. Mode Anal. 117 (1–2), 113–129.
Young, P.C., 2001. Data-based mechanistic modelling and validation of rainfall-flow processes. In: Anderson, M.G., Bates, P.D. (Eds.), Model Validation: Perspectives in Hydrological Science. J. Wiley, Chichester, pp. 117–161.
Young, P.C., 2006. Data-based mechanistic modelling and river flow forecasting. In: 14th IFAC Symposium on System Identification, pp. 756–761. Newcastle, Australia.
Young, P., Garnier, H., 2006. Identification and estimation of continuous-time, data-based mechanistic (DBM) models for environmental systems. Environ. Model. Softw. 21 (8), 1055–1072.
Young, P.C., Jakeman, A., 1980. Refined instrumental variable methods of recursive time-series analysis, part III. Extensions. Int. J. Control 31 (4), 741–764.
Young, P.C., Sumisławska, M.A., 2012. Control systems approach to input estimation with hydrological applications. In: 16th IFAC Symposium on System Identification. July 2012, Brussels, Belgium, pp. 1043–1048.
Young, P.C., Pedregal, D.J., Tych, W., 1999. Dynamic harmonic regression. J. Forecast. 18 (6), 369–394.
Yu, B., Ciesiolka, C., Rose, C., Coughlan, K., 1997. Note on sampling errors in the rainfall and runoff data collected using tipping bucket technology. Trans. ASAE 40 (5), 1305–1309.