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# A Physically Based Model of Heterogeneous Hillslopes
# 2. Effective Hydraulic Conductivities

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Using the results of a fully three-dimensional model of variably saturated flow on a heterogeneous hillslope, the concept of equivalent homogeneous hillslopes is explored. By considering single realizations of random patterns of saturated hydraulic conductivity, attempts are made to determine single effective hydraulic conductivity values capable of reproducing both subsurface and surface flow hydrographs. For the case of high-permeability soils, effective parameters were found to reasonably reproduce the hillslope hydrograph, although there was no consistent relationship between the effective values and the moments of the spatial distributions. For the case of low-permeability soils, characterized by surface flow domination of the runoff hydrograph, single effective parameters were not found to be capable of reproducing both subsurface and surface flow responses. Furthermore, the event dependency of effective conductivity values under such conditions was demonstrated.

## INTRODUCTION

In the first part of this paper [*Binley et al.*, this issue] (hereinafter referred to as part 1) the effects of different random patterns of saturated hydraulic conductivity on a 150 m by 100 m hillslope were investigated. The results demonstrated that, particularly for the case of low conductivity soils producing surface runoff, soil property spatial variability at the hillslope scale may have considerable effect on the runoff hydrograph.

The concept of equivalent parameter values in reproducing the effects of soil spatial variability in current physically based models of catchment hydrology is of fundamental importance. Current hydrologic practice assumes that large-scale heterogeneity can be lumped into effective parameter values (see part 1 for discussion). If such equivalent properties cannot be found to represent real heterogeneous systems, then physically based models have limited practical predictive value. The objective of this study is to examine the validity of effective parameter values for a number of the hydraulic conductivity distributions considered in part 1.

The concept of effective hydraulic conductivity values has been a source of interest for many years. *Cardwell and Parsons* [1945] considered the case of uniform two-dimensional steady saturated flow through a block of porous media made up of smaller blocks of different conductivities. It is easily shown that for an arrangement of blocks in series the harmonic mean ($\hat{K}_H$) of the block values is an equivalent hydraulic conductivity. Similarly, the arithmetic mean ($\hat{K}_A$) represents a system of blocks in parallel. Cardwell and Parsons were able to demonstrate that the effective conductivity for any assemblage of blocks lies between these two extreme values. On the basis of a similar arrangement of permeability cells, *Marshall* [1962] developed an expression for the equivalent permeability of a heterogeneous medium.



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The results of the Monte Carlo simulations of *Warren and Price* [1961], on steady state groundwater flow, led them to suggest that the geometric mean ($\hat{K}_G$) of the individual block hydraulic conductivities is capable of representing the nonuniform system. *Freeze* [1975] disputed these claims in his analysis of transient one-dimensional saturated flow and concluded that, under such conditions, an equivalent uniform media is undefinable.

Using analytical expressions, *Gutjhar et al.* [1978] and *Dagan* [1979] found that for two-dimensional steady saturated flow in domains of low variability the geometric mean of the log normal distribution of hydraulic conductivity is a suitable effective value. However, *Dagan* [1979] estimated the effective value for three-dimensional flow through highly variable soil to be over 4 times larger than $\hat{K}_G$. On the basis of a Monte Carlo analysis of two-dimensional steady saturated flow, *Smith and Freeze* [1979] have shown that the geometric mean effective value may only be suitable under strict conditions.

In their study of vertical infiltration, *Dagan and Bresler* [1983] and *Bresler and Dagan* [1983] demonstrated that effective parameters may only be meaningful under certain restrictive conditions such as steady flow. Difficulties in selecting an equivalent porous medium under conditions of unsteady vertical infiltration are also reported in the earlier work of *Russo and Bresler* [1981].

*Yeh et al.* [1985] presented analytical expressions for the effective conductivity of three-dimensional steady unsaturated flow. For soils of low variability, $\hat{K}_G$ appeared to be a suitable equivalent parameter. Larger values, however, were found to be appropriate for fields of greater nonhomogeneity. The findings of Yeh et al. thus complement the earlier conclusions of *Dagan* [1979] with respect to saturated groundwater movement.

In a Monte Carlo analysis of the drainage of an unconfined aquifer, using a two-dimensional saturated flow approximation, *El-Kadi and Brutsaert* [1985] noted that the effective

TABLE 1. Summary of Findings of Several Previous Studies of Effective Conductivities

| Source | Flow System | Effective Hydraulic Conductivity, $K_{eff}$ |
|---|---|---|
| *Cardwell and Parson* [1945] | two-dimensional steady saturated | $K_{eff} \approx \bar{K}_G$ |
| *Warren and Price* [1961] | three-dimensional steady saturated (low variability) | $K_{eff} \approx \bar{K}_G$ |
| *Gutjhar et al* [1978] | two-dimensional steady saturated | $K_{eff} = \bar{K}_G$ |
| | three-dimensional steady saturated | $K_{eff} \geq \bar{K}_G$ |
| *Dagan* [1979] | two-dimensional steady saturated | $K_{eff} = \bar{K}_G$ |
| | three-dimensional steady saturated | $K_{eff} \geq \bar{K}_G$ |
| *Smith and Freeze* [1979] | two-dimensional uniform steady saturated | $K_{eff} = \bar{K}_G$ |
| *Yeh et al.* [1985] | three-dimensional steady unsaturated | $K_{eff} \geq \bar{K}_G$ |
| *Freeze* [1975] | one-dimensional unsteady saturated | not definable |
| *Russo and Bresler* [1981] | one-dimensional unsteady unsaturated | not definable |
| *Bresler and Dagan* [1983] | one-dimensional unsteady unsaturated | not definable |
| *El-Kadi and Brutsaert* [1985] | two-dimensional unsteady saturated (at large times) | $K_{eff} = \bar{K}_G$ |
| *Dagan* [1982] | three-dimensional unsteady saturated (at large times) | $K_{eff} \geq \bar{K}_G$ |

hydraulic conductivity was a function of time. For small times, using the geometric mean as an equivalent parameter, the outflow of the aquifer was consistently underestimated, although $\bar{K}_G$ was found to be suitable for large times. Variability of effective parameters for unsteady saturated flow had been suggested by *Dagan* [1982], using analytical expressions resulting from perturbation analysis. Dagan had concluded that, for the case of constant head initial conditions, the initial effective conductivity is equal to the arithmetic mean of the log normal distribution. After some time, termed the relaxation time, the effective conductivity is reduced to a value corresponding to that at steady state. Example calculations by *Dagan* [1982] showed that typical values of the relaxation time for three-dimensional groundwater flows are likely to be if the order of several minutes.

Using spectral analysis applied to linearized partial differential equations of soil moisture movement, *Mantoglou and Gelhar* [1987*a*] developed a stochastic theory of three-dimensional transient unsaturated flow. In a study of effective hydraulic conductivity for flow in stratified soils using this model, *Mantoglou and Gelhar* [1987*b*] demonstrated significant hysteresis in the effective values. Such hysteresis was produced by the soil spatial variability rather than the hysteresis of the local parameter values.

A summary of the findings of the several studies discussed is presented in Table 1. The inequality time (>) shown in this table refers to cases of high variability. The geometric mean of the log normal distribution appears to be a suitable estimate of an effective parameter for most systems examined. Difficulties in determining a single equivalent uniform medium for unsteady flows are apparent, particularly in unsaturated media.

## Applicability of Effective Hydraulic Conductivity Values for Hillslope Runoff Generation

In order to assess the validity of effective hydraulic conductivity values for hillslope runoff generation the following two-part procedure was adopted:

1. Taking a single realization of soil spatial variability from the cases described in part 1, attempt to select a suitable equivalent uniform porous medium for a given event.
2. Changing the initial conditions and event, determine whether the selected effective hydraulic conductivity remains valid.

It is important to note that this study aims to find the effective parameters for single realizations of a spatial random conductivity field. Previous studies, such as those discussed earlier, have addressed the problem of effective parameters to describe the average response of a given system, that is, the expected result. In practice it is this single realization which we wish to model, since it is representative of a single real hillslope or aquifer [*Beven*, 1981]. In part 1 we demonstrated that increasing the variance of a conductivity field has an effect on the subsurface hydrograph equivalent to increasing the mean hydraulic conductivity. Therefore the effective conductivity for a collection of multiple samples is slightly larger than the geometric mean of the hydraulic conductivities (given by the population mean), thus complementing the findings of several multidimensional investigations. This result is not strictly true, however, for the total flow hydrograph under conditions of infiltration excess runoff, as increasing the extent of variability increases the amount of surface runoff (Table 1, part 1), thus lowering the effective conductivity. This finding will be readdressed later.

The event used for the first part of the investigation procedure (event 1) is identical to that used in part 1. For the second part of the experiment the initial conditions are described by the steady state solution obtained by prescribing the pressure head at the soil surface as a linear decrease from −90 cm water at the base of the slope to −340 cm water at the top of the slope. The deterministic rainfall event consists of 10 hours of drainage, which is followed by 5 hours of rainfall at an intensity of 3 mm/h, followed by 3 hours of 8 mm/h which precedes 12 hours of drainage. The total rainfall of this second event is thus 39 mm, which is slightly greater than that of event 1 (32 mm), which together with the wetter antecedent conditions of event 2 leads to the greater likelihood of the onset of surface runoff in all soils examined.

Although the equivalent porous medium simulations do not require three-dimensional analysis, since the movement of water occurs only in a two-dimensional plane, the computing cost prohibited the use of an efficient search algorithm to determine the "best" effective parameter. Instead, the volume of the runoff hydrograph was taken as the output variable to be matched and, after selecting an approximate effective conductivity, using the results of over 100 two-dimensional homogeneous soil simulations, a limited search of the parameter space was made.

TABLE 2. Effective Hydraulic Conductivities, Hydrograph Matching Errors, and Sample Properties for Selected Realizations From Cases (B-N) in Part 1

| | | | | | Event 1 | | | | Event 2 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Subsurface Flow | | Total Flow | | Subsurface Flow | | Total Flow | |
| Case | exp ($\mu_Y$), cm/min | $\sigma_Y^2$, ln (cm/min)$^2$ | $K_{eff}$, cm/min | $p$ | Peak Error, % | Volume Error, % | Peak Error, % | Volume Error, % | Peak Error, % | Volume Error, % | Peak Error, % | Volume Error, % |
| B1 | 0.05 | 0.25 | 0.05197 | 0.28 | 0.887 | 0.042 | 1.421 | 0.405 | −0.867 | −0.255 | 0.367 | −0.439 |
| E1 | 0.10 | 0.50 | 0.10304 | 0.11 | 4.495 | 0.007 | 14.264 | 0.185 | 0.419 | 0.315 | 0.009 | −0.567 |
| E2 | 0.10 | 0.50 | 0.11118 | 0.41 | −0.360 | 0.022 | 0.575 | 0.032 | 1.514 | 0.007 | 1.730 | −0.582 |
| E3 | 0.10 | 0.50 | 0.09891 | 0.05 | 3.663 | 0.007 | 12.908 | 0.362 | −0.264 | 0.554 | 1.382 | −0.004 |
| F1 | 0.10 | 1.00 | 0.11929 | 0.35 | 0.314 | −0.054 | 1.285 | −0.039 | 2.043 | −0.229 | −0.362 | −0.764 |
| F2 | 0.10 | 1.00 | 0.10055 | 0.01 | 6.537 | 0.009 | 21.094 | 0.498 | 0.223 | 0.903 | 2.589 | 0.726 |
| H1 | 0.20 | 0.25 | 0.21323 | 0.50 | −0.325 | −0.011 | −0.325 | −0.012 | 1.218 | −0.294 | −1.419 | −0.907 |
| I1 | 0.20 | 0.50 | 0.22159 | 0.40 | 0.886 | 0.004 | 0.886 | 0.004 | 1.796 | −0.416 | 3.170 | −1.025 |
| I2 | 0.20 | 0.50 | 0.19948 | −0.01 | 1.649 | 0.006 | 1.649 | 0.006 | 0.397 | 0.500 | −0.832 | 0.815 |
| J1 | 0.10 | 0.25 | 0.09358 | −0.15 | 0.281 | −0.020 | −1.700 | 1.454 | 0.883 | 1.395 | 0.339 | 0.878 |
| J2 | 0.10 | 0.25 | 0.11882 | 2.30 | 0.815 | −0.007 | 1.127 | −0.004 | 1.021 | 0.350 | 0.657 | −0.072 |
| K1 | 0.10 | 0.50 | 0.09412 | 0.04 | 1.219 | −0.024 | 3.258 | 2.423 | 2.749 | 2.334 | −0.165 | 1.305 |
| K2 | 0.10 | 0.50 | 0.13051 | 1.73 | 1.874 | 0.011 | 1.874 | 0.011 | 1.744 | 0.609 | 0.250 | −0.736 |
| M1 | 0.20 | 0.25 | 0.31794 | 3.24 | 0.374 | −0.034 | 0.374 | −0.034 | 0.073 | −1.213 | 0.782 | −1.699 |
| N1 | 0.20 | 0.50 | 0.39065 | 2.35 | 0.329 | 0.035 | 0.329 | 0.035 | −2.767 | −1.868 | −2.670 | −2.741 |
| N2 | 0.20 | 0.50 | 0.18914 | 0.07 | 2.040 | −0.022 | 6.202 | 0.171 | 2.991 | 1.535 | −3.409 | 2.246 |

### *Effective Parameters for High-Permeability Soils*

A total of 16 realizations from cases B to N in part 1 were selected in order to determine suitable effective parameters. Since in all these cases overland flow (by surface saturation) was seen to occur during the latter stage of the rainfall event, it was decided to attempt to match the subsurface flow volume of runoff and then determine the suitability of the effective conductivity in reproducing the peak total (subsurface plus surface) flow. The "best" parameter values are shown in Table 2, together with the population statistics of the various cases. In this table, E1 refers to example 1 from case E, E2 to example 2, and so on. The error of each effective parameter simulation was evaluated by four quantities: subsurface flow peak and volumetric errors and total flow peak and volumetric errors. Each error was expressed as a percentage using

$$\text{Percentage error} = 100\% \times (f - f_{eff}/f)$$

where $f$ is the peak or volume of flow produced by the

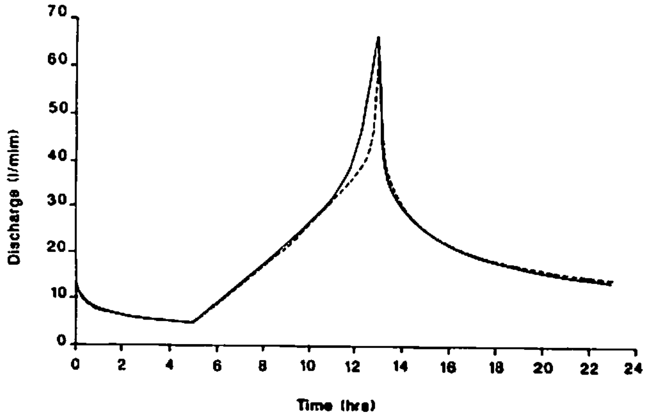


Fig. 1. Total flow hydrographs for heterogeneous and homogeneous hillslopes. Solid line indicates results from case K1, event 1. Dashed line indicates effective parameter solution.

nonuniform case and $f_{eff}$ is the corresponding value for the uniform simulation. Table 2 columns 6 to 9 show the matching errors of the event 1 simulations using the effective parameter values in column 4. It can be seen from the values that, for most cases, matching the volume of subsurface runoff produces a reasonable effective conductivity for the total flow hydrograph. The effective parameters for the uncorrelated realizations E1, E3, and F2 show the greatest errors in reproducing the peak total flow. These three samples are taken from populations with mean values of 0.1 cm/min, which is approximately equal to a "surface saturation threshold conductivity" for this particular event and slope, that is, conductivity values greater than 0.1 cm/min will not produce surface saturation. Therefore as the nonuniformity of the soil properties tends to increase the value of an equivalent conductivity for subsurface movement, if that value is greater than 0.1 cm/min, any likelihood of surface saturation is removed.

It is clear from the results in Table 2 that increasing the variability of the field reduces the applicability of the selected effective parameters, albeit to a minor extent. The total flow hydrograph for case K1 with the corresponding responses of the uniform cases are shown in Figure 1. It can be seen from this diagram that the equivalent parameter value is suitable throughout most of the event, any discrepancy occuring at the hydrograph peak. Adopting the effective conductivities for event 1, the matching errors for event 2 are shown in Table 2 columns 10 to 13. The relatively low percentage errors displayed suggest that the equivalent medium properties selected for event 1 remain suitable under the entirely different conditions of event 2. A comparison of the total flow hydrograph for case N2 and the selected effective parameter value is shown in Figure 2. As before, the effective value appears suitable throughout much of the event, in particular during the drying stages. In order to demonstrate clearly the relationship between the effective parameter found in this way and the underlying distribution parameters the following equation was used:

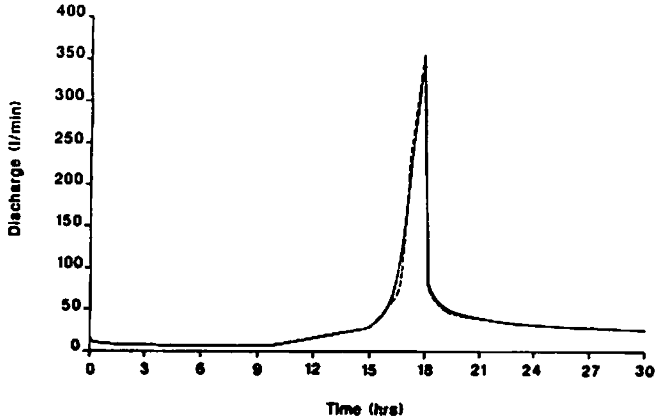


Fig. 2. Total flow hydrographs for heterogeneous and homogeneous hillslopes. Solid line indicates results from case N2, event 2. Dashed line indicates effective parameter solution.

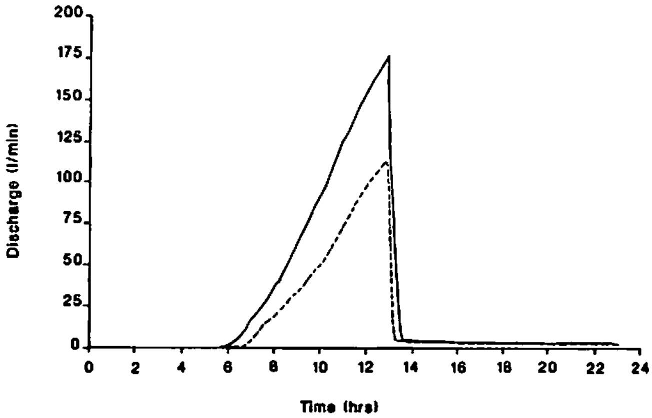


Fig. 4. Total flow hydrographs for heterogeneous and homogeneous hillslopes. Solid line indicates results from case S2, event 1. Dashed line indicates effective parameter solution.

$$K_{\text{eff}} = \bar{K}_G \exp \frac{\{p\sigma^2\}}{2} \quad (1)$$

where $\sigma^2$ is the variance of ln $(K_s)$ and $p$ is a scaling parameter. Values of $p$ equal to $-1$, 0, and 1 result in (1) being equal to the harmonic, geometric, and arithmetic means, respectively. The magnitude of $p$ for the given effective conductivity for each of the 16 stochastic fields is shown in Table 2 column 5. For most cases the effective parameter ($K_{\text{eff}}$) is greater than the geometric mean of the sample distributions, thus complementing the findings of previous studies on simpler flow systems. In four cases (J2, K2, M1, and N1) the effective conductivity is greater than the arithmetic mean ($p = 1$). These four cases are examples of highly correlated distributions and demonstrate the effect of large areas of high conductivity near the base of the slope on the resulting equivalent parameter value. Figure 3 shows plots of subsurface flow volume discrepancy against $p$ for cases B1, F1, I1, M1, and N1. The intersect of the $p$ axis is noticeably similar for the uncorrelated cases (B1, F1, and I1) and much less than that for the correlated cases. *Gomes-Hernandez and Gorelick* [1987], using an expression equivalent to (1), reported values of $p$ equal to $-0.4$ to provide suitable aquifer effective conductivities. The range of values in Table 2 show much greater deviations from the geometric mean.

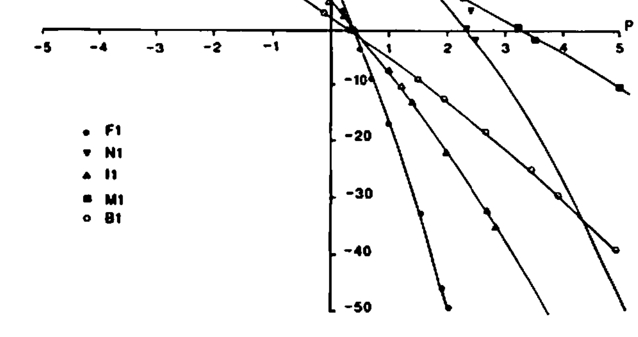


Fig. 3. Variation of subsurface flow volume discrepancy expressed as a percentage against parameter $p$ of (1) for cases B1, F1, I1, M1, and N1.

## *Effective Parameters for Low Permeability Soils*

Five realizations from cases P to S in part 1 were selected for analysis of effective parameters under conditions of infiltration excess runoff. Adopting the same procedure as the previous section on high-permeability soils, attempts were first made to determine effective values capable of reproducing the volume of subsurface flow. However, since in the examples presented in this section overland flow dominates the hydrograph, the procedure was repeated using the volume of the total flow hydrograph as the matching output variable.

The effective hydraulic conductivities for the five cases are shown in Table 3, together with the population statistics of the conductivity fields. Within this section, P1 refers to example 1 of case P, using the subsurface flow hydrograph volume as the output variable of interest as before, and PP1 refers to the same case analyzed using the total flow hydrograph volume. It can be seen from the values of $K_{\text{eff}}$ in Table

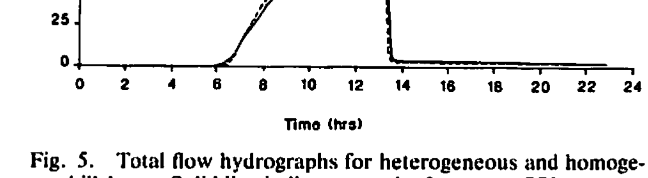


Fig. 5. Total flow hydrographs for heterogeneous and homogeneous hillslopes. Solid line indicates results from case SS2, event 1. Dashed line indicates effective parameter solution.

TABLE 3. Effective Hydraulic Conductivities, Hydrograph Matching Errors, and Sample Properties for Selected Realizations From Cases (P-S) in Part 1

| | | | | | Event 1 | | | | Event 2 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Subsurface Flow | | Total Flow | | Subsurface Flow | | Total Flow | |
| Case | $\exp(\mu_Y)$, cm/min | $\sigma_Y^2$ ln (cm/min)$^2$ | $K_{eff}$, cm/min | $p$ | Peak Error, % | Volume Error, % | Peak Error, % | Volume Error, % | Peak Error, % | Volume Error, % | Peak Error, % | Volume Error, % |
| P1 | 0.005 | 0.50 | 0.00546 | 0.34 | 1.634 | −0.788 | 14.604 | 16.563 | 2.331 | −0.081 | 5.581 | 9.943 |
| R1 | 0.005 | 0.25 | 0.00760 | 2.85 | −1.821 | −0.696 | 13.428 | 16.172 | −1.077 | 0.559 | 36.137 | 44.473 |
| R2 | 0.005 | 0.25 | 0.00411 | 0.17 | −1.172 | 0.395 | 28.494 | 29.305 | −8.875 | 1.289 | −1.069 | 3.732 |
| S1 | 0.005 | 0.50 | 0.00908 | 2.04 | −3.160 | −0.239 | 23.198 | 23.784 | −0.180 | 1.817 | 47.326 | 57.771 |
| S2 | 0.005 | 0.50 | 0.00380 | 0.13 | −6.621 | 0.594 | 34.724 | 41.115 | −27.381 | 0.437 | −0.891 | 4.487 |
| PP1 | 0.005 | 0.50 | 0.00465 | −0.31 | 16.236 | 15.115 | 7.477 | 1.723 | 16.820 | 14.842 | −8.243 | −13.894 |
| RR1 | 0.005 | 0.25 | 0.00480 | −1.11 | 35.954 | 34.572 | 0.215 | −1.040 | 36.082 | 35.932 | −14.202 | −19.643 |
| RR2 | 0.005 | 0.25 | 0.00342 | −1.59 | 16.140 | 20.710 | 4.585 | 2.589 | 9.662 | 20.771 | −12.745 | −15.517 |
| SS1 | 0.005 | 0.50 | 0.00445 | −1.04 | 49.242 | 48.040 | 3.597 | 0.774 | 50.921 | 50.230 | −23.309 | −31.377 |
| SS2 | 0.005 | 0.50 | 0.00290 | −1.16 | 18.716 | 30.104 | 1.297 | 3.693 | 41.519 | 32.336 | −15.420 | −20.353 |

3 that there is considerable difference between the two effective parameters of a given field.

The matching errors for the 10 effective conductivity values for event 1 are displayed in Table 3 columns 6 to 9. The difficulty of reproducing both subsurface flow and total flow hydrographs using a single equivalent porous medium can be seen from the large error reported in this table. The magnitudes of errors are much greater than those for high-permeability soils in Table 2.

The different results of the two types of effective parameters are demonstrated further in Figures 4 and 5 for cases S2 and SS2, respectively. Matching the subsurface flow in Figure 4 clearly fails to reproduce the time of onset and rate of surface runoff. The underestimation of subsurface flow rates caused by reducing the effective conductivity to produce greater overland flow volumes can be seen clearly in the hydrograph tail for case SS1 in Figure 6.

Using the same effective parameters, the matching errors for event 2 are shown in Table 3 columns 10 to 13. The suitability of reproducing the volume of subsurface flow for cases P1 to S2 is indicated by the low errors reported. However, the effective conductivities for cases PP1 and SS2 fail to provide reasonable estimates of subsurface and surface flow quantities. In these cases the subsurface flow hydrograph is consistently underestimated, and the surface flow hydrograph is consistently overestimated. Furthermore, the greater flow rates produced in event 2 imply a greater significance of the percentage errors in Table 3. This overestimation of total flow is shown clearly in Figure 7 for case SS2.

The overestimation of surface runoff using effective hydraulic conductivities determined from a lower intensity storm can be explained by considering the range of conductivities, in a given distribution, which contribute to the generation of overland flow. For event 1 the effective parameter value will be controlled by that part of the conductivity distribution producing overland flow. For a higher intensity storm (event 2) the range of contributing permeability values will increase. Thus the effective conductivity will be underestimated for event 2, resulting in an overestimation of surface runoff.

The magnitude of $p$ in (1) for each of the individual realizations from cases P to S is shown in Table 3 column 5. As in the case of high-permeability soils, an effective hydraulic conductivity greater than the geometric mean of the distribution is required to reproduce the subsurface flow hydrograph volumetrically. However, in order to estimate the total flow hydrograph, an effective parameter much less than the geometric mean is required. Consequently, use of a single effective parameter value is not justified. This is

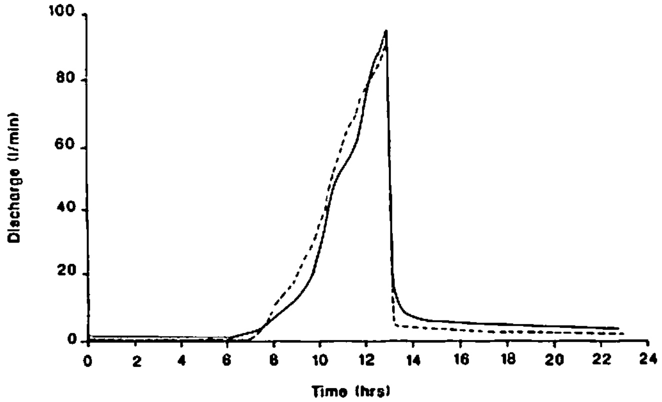


Fig. 6. Total flow hydrographs for heterogeneous and homogeneous hillslopes. Solid line indicates results from case SS1, event 1. Dashed line indicates effective parameter solution.

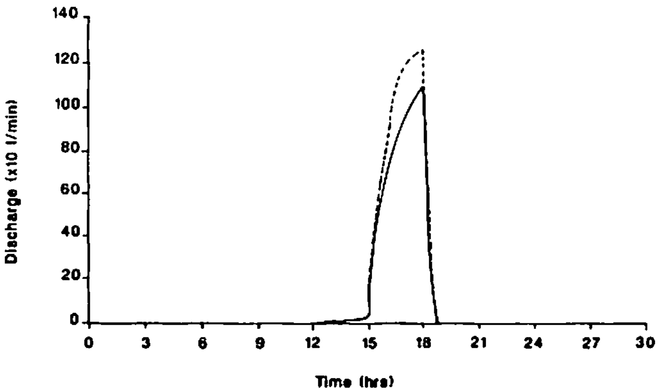


Fig. 7. Total flow hydrographs for heterogeneous and homogeneous hillslopes. Solid line indicates results from case SS2, event 2. Dashed line indicates effective parameter solution.

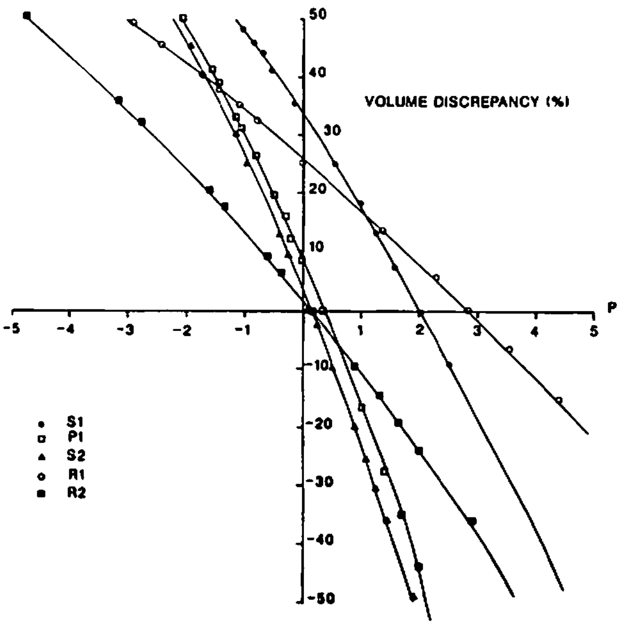


Fig. 8. Variation of subsurface flow volume discrepancy expressed as a percentage against parameter $p$ of (1) for cases P1, R1, R2, S1, and S2.

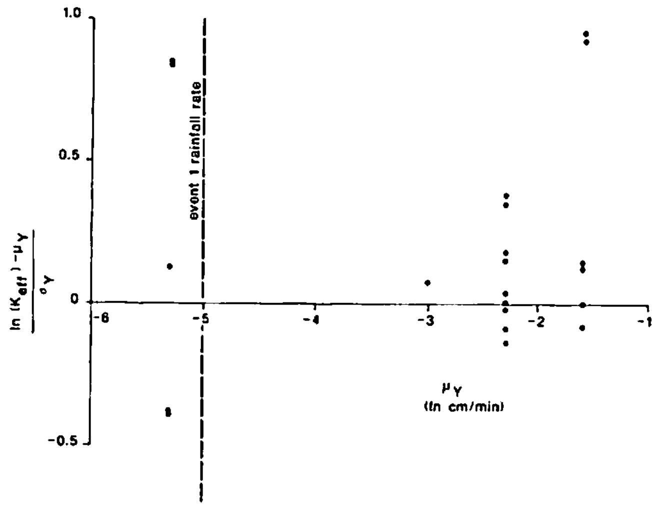


Fig. 10. Variation of effective hydraulic conductivity with population mean conductivity for all 21 realizations.

further demonstrated by comparing Figures 8 and 9, which show plots of subsurface flow and total flow volume discrepancies against parameter $p$ of (1), respectively, for cases P1 to SS2.

In Figure 10 the subsurface flow effective values for all 21 realizations considered (16 high-permeability, 5 low-permeability) are shown on a normalized log scale. For the high-permeability case the range of effective conductivity values is shown to increase with increasing population mean. However, when the mean conductivity is below the rainfall rate, the range of effective values is also high. The determination of effective parameter values, based on population statistics, is clearly not a straightforward (if possible) task.

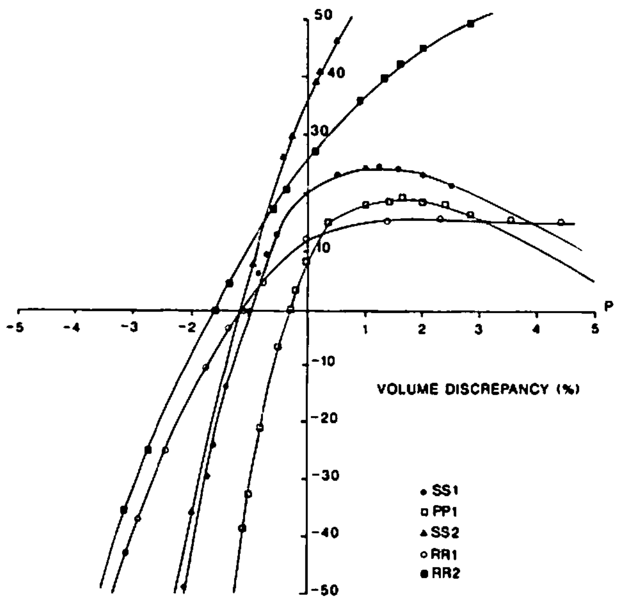


Fig. 9. Variation of total flow volume discrepancy expressed as a percentage against parameter $p$ of (1) for cases PP1, RR1, RR2, SS1, and SS2.

## Summary and Conclusions

The majority of previous studies of effective parameters have directed attention to relatively simple flow systems, such as steady saturated groundwater movement. Investigations of three-dimensional transient variably saturated flow in porous media are not evident in the literature. Furthermore, analyses of hillslope flow processes using effective parameters are scarce, although the results of such studies are clearly important if current physically based modeling techniques are to be recognized as practical tools.

Using the results from various hydraulic conductivity fields presented in part one of this paper [*Binley et al.*, this issue], attempts were made to determine suitable equivalent saturated hydraulic conductivities capable of reproducing the response to a single deterministic event. In the case of high-permeability soils, effective parameters were found to give reasonable estimates under a variety of hydraulic conductivity fields. These values were also shown to be suitable for a completely different event. For low-permeability soils, characterized by the domination of the hydrograph by surface runoff, a single effective parameter was not found to be capable of reproducing both subsurface and surface flow hydrographs. Matching the subsurface flow, in this case, consistently underestimated the surface runoff; matching the total flow hydrograph underestimated the subsurface flow response. Application of the selected effective parameters to a second event revealed the event dependency of the equivalent properties.

For all subsurface flow hydrographs there appeared to be little time dependency of the effective parameters. This is consistent with the low relaxation time suggested by *Dagan* [1982] for three-dimensional systems.

The results suggest that the subsurface response of heterogeneous hillslopes may be reasonably represented using effective hydraulic conductivities. The value of this equivalent property may not be obtained explicitly from the statistics of the distribution (even if these are known a priori), although it is likely to be greater than the geometric mean of the log normally distributed conductivity field. In the case of low-permeability soils, single effective parameter values appear invalid for modeling both flow processes.

A possible solution to this problem might be to form a composite hydrograph from a number of simulations with different effective parameters. For example, a linear combination of the results of two independent simulations, obtained from uniform soils of different properties, is likely to produce less error in the predicted overall response than using a single simulation. If the nature of the spatial variability were known or could be assessed a priori, then suitable properties of the different uniform media may be selected from the characteristics of the assumed probability distribution. The combination of the multiple simulations would clearly require calibration from previous records. In addition, stationarity of the combination function is necessary if future predictions are to be made. Such an approach is currently being investigated, the results of which will be reported in the near future.

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## REFERENCES

Beven, K., Comment on 'A stochastic-conceptual analysis of rainfall-runoff processes on a hillslope' by R. Allan Freeze, *Water Resour. Res.*, *17*(2), 431–432, 1981.

Binley, A., J. Elgy, and K. Beven, A Physically based model of heterogeneous hillslopes, 1, Runoff production, *Water Resour. Res.*, this issue.

Bresler, E., and G. Dagan, Unsaturated flow in spatially variable fields, 2, Application of water flow models to various fields, *Water Resour. Res.*, *19*(2), 421–428, 1983.

Cardwell, W. L., and R. L. Parsons, Average permeabilities of heterogeneous oil sands, *Trans. Am. Inst. Min. Metall. Pet. Eng.*, *160*, 34–43, 1945.

Dagan, G., Models of groundwater flow in statistically homogeneous porous formations, *Water Resour. Res.*, *15*(1), 47–63, 1979.

Dagan, G., Analysis of flow through heterogeneous random aquifers, 2, Unsteady flow in confined aquifers, *Water Resour. Res.*, *18*(5), 1571–1585, 1982.

Dagan, G., and E. Bresler, Unsaturated flow in spatially variable fields, 1, Derivation of models of infiltration and redistribution, *Water Resour. Res.*, *19*(2), 413–420, 1983.

El-Kadi, A., and W. Brutsaert, Applicability of effective parameters for unsteady flow in nonuniform aquifers, *Water Resour. Res.*, *21*(2), 183–198, 1985.

Freeze, R. A., A stochastic-conceptual analysis of one-dimensional groundwater flow in a nonuniform homogeneous media, *Water Resour. Res.*, *11*(5), 725–741, 1975.

Gomez-Hernandez, J. J., and S. M. Gorelick, Effect of spatial variability of hydraulic conductivity, leakance and recharge in a realistic aquifer: Is there an effective permeability?, paper presented at the I.A.S.H. Workshop on Spatial Variability and representativeness of hydrogeological parameters, Int. Union Geod. and Geophys., General Assembly, Vancouver, Canada, August 11–12, 1987.

Gutjhar, A. L., L. W. Gelhar, A. A. Bakr, and J. L. MacMillan, Stochastic analysis of spatial variability in subsurface flows, 2, Evaluation and application, *Water Resour. Res.*, *14*(5), 953–959, 1978.

Mantoglou, A.,and L. W. Gelhar, Stochastic modeling of large-scale transient unsaturated flow systems, *Water Resour. Res.*, *23*(1), 37–46, 1987*a*.

Mantoglou, A., and L. W. Gelhar, Effective hydraulic conductivities of transient unsaturated flow in stratified soils, *Water Resour. Res.*, *23*(1), 57–67, 1987*b*.

Marshall, T. J., Permeability equations and their models, in *Proceedings of the Symposium on Interaction between Fluids and Particles*, pp. 299–303, Institute of Chemical Engineers, London, 1962.

Russo, D., and E. Bresler, Effect of field variability in soil hydraulic properties on solutions of unsaturated soil and water flows, *Soil Sci. Soc. Am. J.*, *45*, 675, 681, 1981.

Smith, L., and R. A. Freeze, Stochastic analysis of steady state groundwater flow in a bounded domain, 2, Two-dimensional simulations, *Water Resour. Res.*, *15*(6), 1543–1559, 1979.

Warren, J. E., and H. S. Price, Flow in heterogeneous porous media, *Soc. Pet. Eng. J.*, *1*, 153–169, 1961.

Yeh, T. C. J., L. W. Gelhar, and A. L. Gutjhar, Stochastic analysis of unsaturated flow in heterogeneous soils, 1, Statistically isotropic media, *Water Resour. Res.*, *21*(4), 447–456, 1985.

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