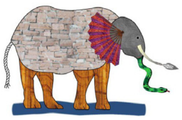

Rosenblueth's and Harr's point estimation methods – Neil McIntyre (Rosenblueth, E. & Harr. M.E.)

PUB-IAHS Workshop

Uncertainty Analysis in Environmental Modelling

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a) Introduction:

Rosenblueth's point estimation method for propagating symmetric and non-symmetric PDFs (Rosenblueth, 1981) aims to reduce the computational demands of propagating uncertainty through a function by eliminating the calculation of derivatives or use of Monte Carlo sampling. The PDF of each random variable is represented by p discrete points, located according to the first, second and third moments of the PDF. The joint PDF of N random variables is represented by an array of projected points, with pN points used (see example below). Each point is assigned a probability mass according to the third moment and the correlation matrix. All points are propagated discretely through the function to pN solutions and the first moment is the weighted average; the second moment, that of the squares; and the third moment, that of the cubes. Most usually a 2-point scheme is used whereby 2N points are required. For symmetrical distributions for N > 2, the number of evaluations can be reduced from 2N to 2N by using Harr's point estimation method (Harr, 1989) by considering eigenvalues. A very useful demonstration of Rosenblueth's method and some discussion of both these methods are given in Christian and Baecher (1999). Protopapas and Bras (1990) have applied Rosenblueth's 2-point method to a rainfall-runoff model and Yeh et al. (1997) have similarly applied Harr's method.

b) Advantages

- Rosenblueth's method is a computationally efficient method of estimating the first 3 moments of functions of a small number of uncertain input variables, which may be correlated and skewed. It has been found to give a good approximation of the first three moments for simple problems (e.g. example below).
- Harr's method is even more efficient for more than 2 uncertain input variables, but is limited to symmetrical distributions.
- Both should provide better estimates than the first order second moment (FOSM) approximation, often without significantly more computation.
- May be easily applied to simulation modelling, given limitations and assumptions below.

c) Disadvantages

Rosenblueth's 2-point method:

- Becomes inefficient when many uncertain inputs are involved (e.g. 20 inputs would result in more than 1 million function evaluations).
- Does not evaluate moments higher than the third, so is likely to be inadequate for evaluating extreme percentiles of hydrologic model output uncertainty.
- Furthermore, Christian and Baecher (1999) note that it is never reliable for moments higher than second, and not generally reliable for non-linear functions with high input variance (however see example below).
- Neglects non-linear inter-dependencies of input variables (i.e. represents them using correlation coefficients)
- Information about input-output sensitivities is not implicit to the method (as it is in FOSM and arguably in Monte Carlo).

Harr's method

- Suffers from the same disadvantages, and is less reliable than Rosenblueth's for skewed input distributions.

d) Assumptions

Uncertainty can be adequately described using lower moments and correlation coefficients.

e) Most appropriate application areas

Where an approximation of output variance is required, rather than probabilistic risk analysis or sensitivity analysis.

Where variance of inputs is not high, or where function is linear or nearly linear.

f) Reading list

Yeh, K., Yang, J.C. and Tung, Y.K. 1997; Regionalization of unit hydrograph parameters; 2. Uncertainty analysis. Stochastic Hydrology and Hydraulics, 11, 173-192.

Protopapas, A.L. and Bras, R.L. 1990; Uncertainty propagation with numerical models for flow and solute transport in the unsaturated zone. Water Resources Research, 26, 10, 2463-2474.

g) Software availability - none

h) Web links or other information - none

i) Figures

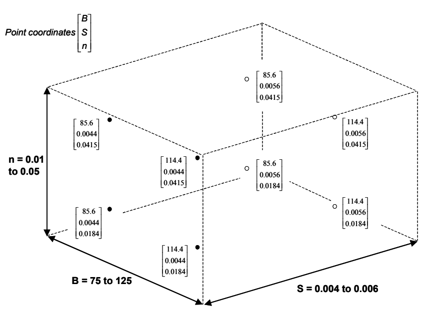

Figure 1 Application of Rosenblueth point estimation to *B*, *S* and *n*

Consider Manning's equation for calculating depth *y* of steady uniform flow in a wide rectangular channel,

$$y = \frac{n^{0.6} Q^{0.6}}{B^{0.6} S^{0.3}}$$

where *y* is the water depth, *Q* is flow (say 100m³/s), *B* is channel width, *S* is channel gradient, *n* is Manning's coefficient. *B*, *S* and *n* are independent random variables with uniform probability distributions (min, max),

B = (75, 125) m
S = (0.004, 0.006)
n = (0.01, 0.05) $m^{-1/3}s$

The uncertainty in *y* is solved using the 2-point method with 8 function evaluations. In this case, as the variables are independent and their distributions are not skewed, each point (see Figure 1) has the same mass *W* = 1/8. Then the moments of *y* are estimated as,

$$M_1(y) = \sum_{i=1}^{8} W_i \frac{n_i^{0.6} Q^{0.6}}{B_i^{0.6} S_i^{0.3}} = 0.594$$

$$M_2(y) = \sum_{i=1}^{8} W_i \left[\frac{n_i^{0.6} Q^{0.6}}{B_i^{0.6} S_i^{0.3}}\right]^2 = 0.377$$

$$M_3(y) = \sum_{i=1}^{8} W_i \left[\frac{n_i^{0.6} Q^{0.6}}{B_i^{0.6} S_i^{0.3}}\right]^3 = 0.252$$

where *i* represents the i^{th} point estimate. Despite the non-linearity of the function, these moments are accurate to 2 decimal places, whereas Monte Carlo requires more than 500 function evaluations to achieve the same precision. However, note the shape of the output PDF derived, for example, using only the lower moments and assuming a normal distribution compared with that using Monte Carlo (Figure 2).

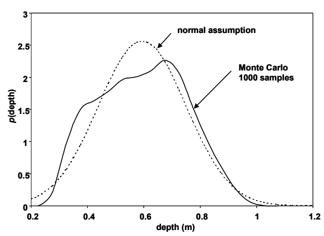

Figure 2 Comparison of 'true' output distribution shape with using lower 2 moments and assuming a normal distribution

j) Delegates Comments (please add !!)